1. Strain Energy Method

→ A structure is a assemblage of members to support external loads.

→ In the process of supporting load, each member is subjected to one or more of the following stress redullants.

i) Axial force → Tensile or compressive force

ii) Bending Moment → Beams

iii) Transverse shear → Beams

iv) Torsion → Ring Beams in RCC, water Tanks.

→ Analysis of structure aims at the determination of the above mensioned stress resultant and deformations which they Produce.

→ Theory of structure results with the rigidity, strength and the stability of engineering structure.

The following clasical methods available for the analysis of structure.

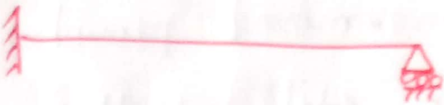
i) Strain Energy Method
ii) Slope Deflection Method
iii) Moment Distribution Method
iv) Column Analogy Method

The application of strain energy method for the analysis of continuous beam, plane rigid frame and statically indeterminate structures or plane truss is briefly deals with this chapter.

Strain Energy:

When a structure is loaded the members are subjected to stresses and strain. The external work done by loads is storeyed as energy due to the straining of the material of the members and hence it is called strain Energy.

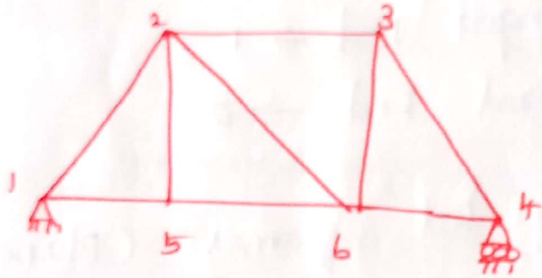
This strain energy in the structure helps the structure to regain its original shape and dimensions when ever the



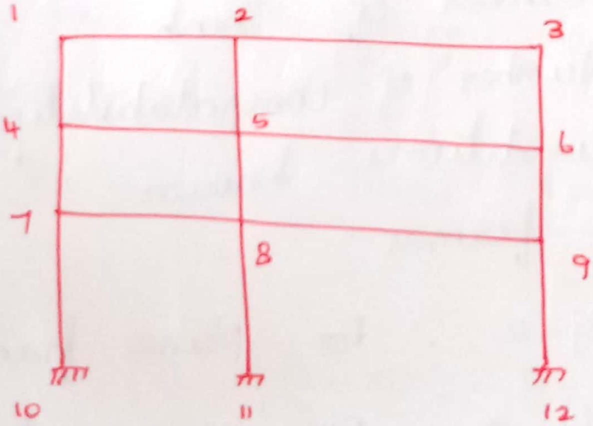
$D_K = 2$

$D_K = 3$

Degree of redundancy for pin jointed Frames



$$D_K = 2j - e$$
$$= 2(6) - 3$$
$$D_K = 9$$



$$D_K = 3j - e$$
$$= 3(12) - 3$$
$$D_K = 27$$

external loads the removed provided the material of the structure is still within the elastic limit.

Degree of Freedom:

i) Free End → 3
ii) Simply supported / roller end → 2
iii) Hinged End → 1
iv) Fixed End → 0

For pin jointed frames: (Plane)

$D_K = 2j - e$

For space frames:

$D_K = 3j - e$

j → Number of Joints
e → Number of compatability (boundary) conditions known

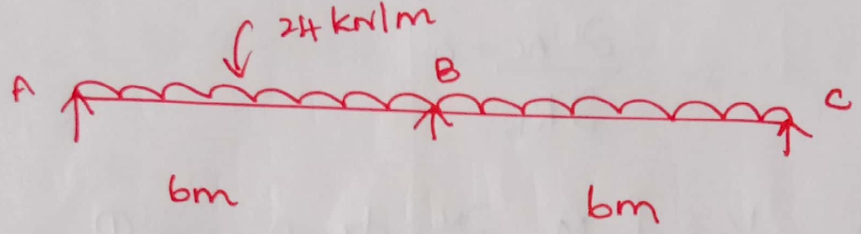
For rigid frames:

$D_K = 3j - e$; For plane frames
$D_K = 6j - e$; For space frames

Degree of Redundancy for beams:

$D_K = 0$

Analyse the continuous beam loaded as shown in the figure by strain energy method. Take EI is constant.



Step 1: To find static Indeterminancy:

static Indeterminancy $= r - e$

$S.I = 3 - 2 = 1$

Hence the beam is statically indeterminate to the find degree. Let us treat R_B as the redundant.

Since the beam and loading are symmetrical.

$$R_A = R_C = \frac{\text{Total load} - R_B}{2}$$

$$= \frac{(24 \times 12) - R_B}{2}$$

$$= 288/2 - \frac{R_B}{2}$$

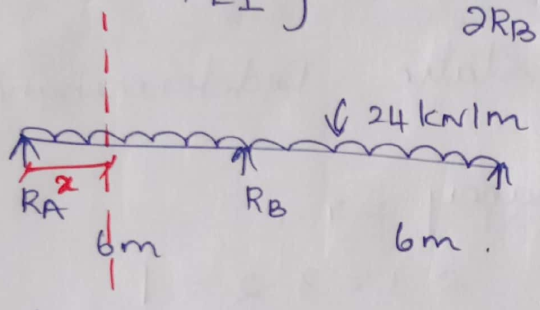
$$R_A = R_C = 144 - 0.5\,R_B$$

The Partial derivative of the total strain energy U in the beam AC with respect to R_B is zero.

since $\delta_B = 0$

$$\frac{\partial U_{AC}}{\partial R_B} = 0$$

ie) $$\frac{1}{EI}\int M \frac{\partial M}{\partial R_B}\, dx = 0 \quad \text{--- (1)}$$



considering a section xx at a distance of x from A in span AB

step 3:

To find moment at x:

considering a section XX at a distance of x from A in span AB

$$M_x = (R_A \times x) - (24 \times x \times x/2)$$

$$= \left[(144 - 0.5R_B)x - 24\, x^2/2\right]$$

$$M_x = 144x - 0.5R_B x - 12x^2 \quad \text{--- (2)}$$

$$\frac{\partial M_x}{\partial R_B} = -0.5x$$

step4: To find Reaction R_B:

The integration limits are 0 and 6m.

$$① \Rightarrow \frac{\partial u}{\partial R_B} \Rightarrow \frac{1}{EI}\int M_x \frac{\partial M_x}{\partial R_B}\, dx = 0$$

$$\Rightarrow \left\{ \frac{1}{EI}\int_0^6 (144x - 0.5R_Bx - 12x^2)(-0.5x)\, dx \right\} x_2 = 0$$

$$\Rightarrow \left\{ \frac{1}{EI}\int_0^6 (144x)(-0.5x)(-0.5R_Bx) - (0.5x)(-12x^2 - 0.5x)\, dx \right\} x_2 = 0.$$

$$\Rightarrow \left\{ \frac{1}{EI}\int_0^6 \left[-72x^2 + 0.25R_Bx^2 + 6x^3\right] dx \right\} x_2 = 0$$

$$\Rightarrow \left\{ \frac{1}{EI}\left[-72\left(\frac{6^3}{3}\right) + 0.25R_B\left(\frac{\cancel{6}^2}{3}\right) + 6\left(\frac{6^3}{3}\right)\right] - (0) \right\} = 0$$

$$\Rightarrow -5184 + 18R_B + 1944 = 0.$$

$$18R_{B2} = 5184 - 1944.$$

$$R_B = 180\,kN$$

step 5: substitute R_B value:

$$② \Rightarrow M_x = 144x - 0.5(180)x - 12x^2$$

$$M_x = 144x - 90x - 12x^2$$

When,

$x = 0$, $M_A = 0$

$x = 6m$; $M_B = 144(6) - 90(6) - 12(6^2)$

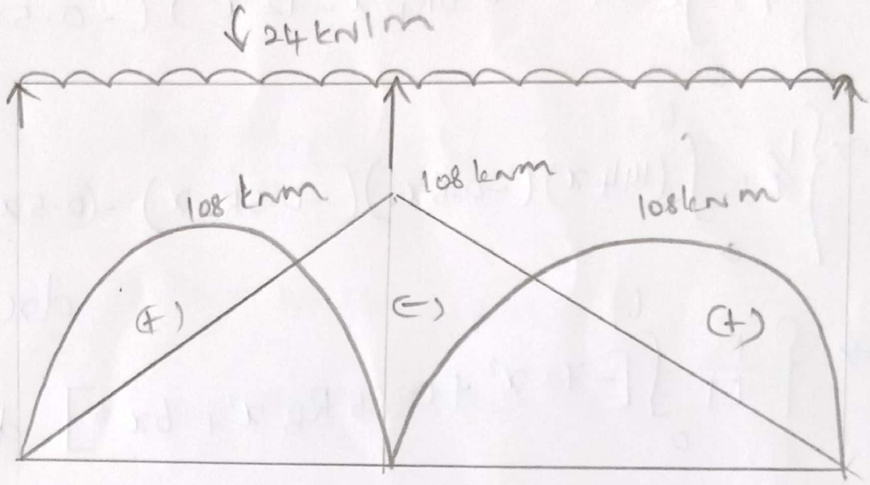
$x = 0$: $M_C = 0$.

$M_B = -108\,kNm$.

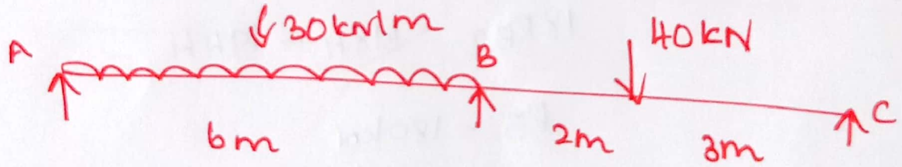
Step 6 : To find Free Bending moment

Span AB $= wL^2/8 = 108$ kN m

Span BC $= wL^2/8 = 108$ kN m



Analyse the continuous beam loaded as shown in figure by strain energy method.



Step 1 : To find degree of static Indeterminancy :

$S \cdot I = r - e$

$= 3 - 2$

$S \cdot I = 1$

Hence the beam is statically indeterminate to the first degree. Let us treat R_B as redundant and Taking moment about c.

Taking moment about "c":

$$(R_A \times 11) + (R_B \times 5) - (30 \times 6) \times (6/2 + 5) - (40 \times 3) = 0$$

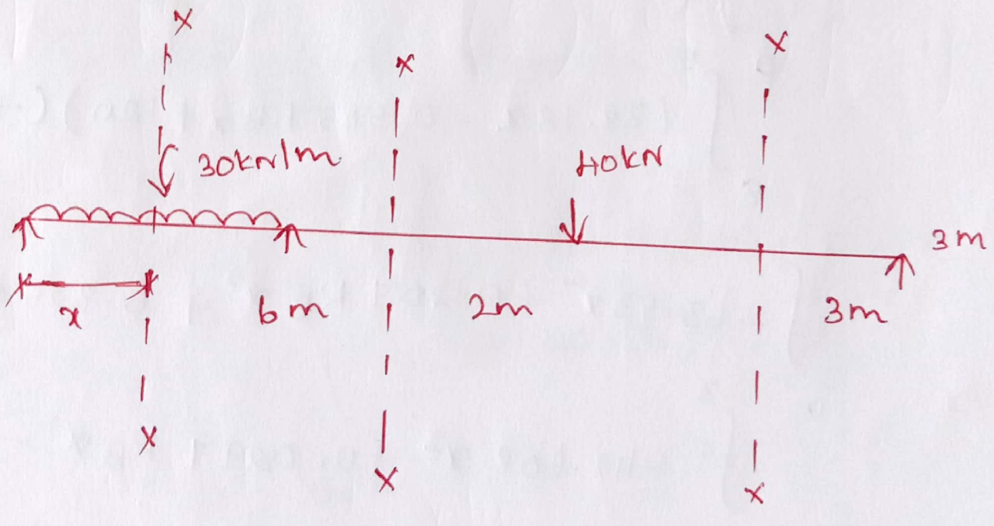
$$R_A = \frac{1560 - 5R_B}{11}$$

$$R_A = \frac{1560}{11} - \frac{5R_B}{11}$$

$$R_A = 141.82 - 0.455 R_B$$

$$R_C = 78.18 - 0.545 R_B$$

step 3 :



$$\frac{\partial U}{\partial R_B} = 0 \quad (\text{since } \delta_B = 0)$$

$$\frac{1}{EI} \int M \frac{\partial M}{\partial R_B} \cdot dx = 0$$

EI is removed and can be removed.
The integration with be done seperately from the three zones AB, CD, DB.

Portion	origin	Limits	M_x	$\partial M_x/\partial R_B$
AB	A	0 to 6	$(1+1.82-0.455R_B)x - \frac{30x^2}{2}$	$-0.455x$
CD	C	0 to 3	$(78.18-0.5+5R_B)x$	$-0.545x$
CB	C	3 to 5	$(78.18-0.548R_B)x-40(x-3)$	$-0.545x$

step 4: To find reactions

substitute the values from the table in equation ① we get;

$$\int_0^6 (141.82-0.455R_B x - 15x^2)(-0.455x)\,dx + \int_0^3 (78.18x - 0.545R_B x)(-0.545x)\,dx$$

$$\int_8^5 (38.18x - 0.545R_B x + 120)(-0.545x)\,dx = 0$$

$$\int_0^6 -63.53x^2 + 0.207R_B x^2 + 6.825x^3)\,dx +$$

$$\int_0^3 -42.608x^2 + 0.0297R_B x^2 - 65.4x)\,dx = 0$$

$$\left[-64.53\,x^3/3 + 0.207R_B\,x^3/3 + 6.825\frac{x^4}{4}\right]_0^6 +$$

$$\left[-42.608\,x^3/3 + 0.297R_B\,x^3/3\right]_0^3 +$$

$$\left[-20.808\,x^3/3 + 0.297R_B\,x^3/3 - 65.4x^2/2\right]_5^3$$

$-46.46 + 14.90\ R_B + 2211.3 - 383.47 + 2.673\ R_B$

$-867 + 12.375\ R_B - 811.5 + 187.27 + 2.612\ R_B = 0$

$\Rightarrow 27.27\ R_B = 40.28 \times 6$

$$R_B = \frac{40.28 \times 6}{27.27}$$

$$R_B = 141.4\ kN$$

$$R_A = -15\ kN$$

$$R_C = -2.19$$

step5: To find final Moments:

when, $x = 0$; $M_A = 0$

$x = 6$; $M_B = 41.82(6) - (0.45 \times 147.40 \times 6) - \frac{30}{2}$

$= 11.5\ kNm$

when, $x = 0$; $M_C = 0$

when $x = M$; $M_D = (78.18 \times 3) - 0.545$

$M_D = -6.5\ kNm$

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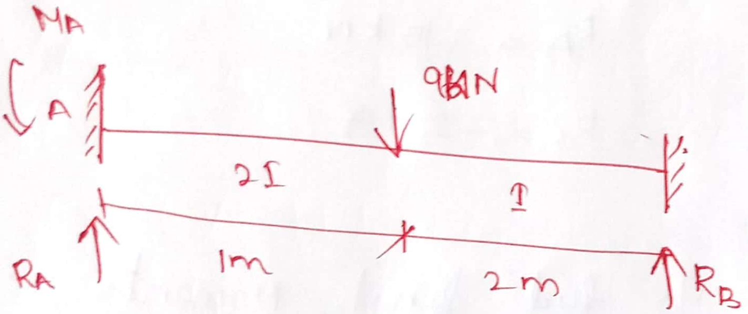
A beam AB of span is fixed at both ends and carries a point load of 9 kN/m² at C. distance 1 m from A. The moment of inertia of the portion AC of the beam is 2I. And that of CD is I. Calculate the fixed end moments and reactions by strain energy Method.



step 1: To find static Indeterminancy:

Degree of redundancy (or) Degree of static Indeterminancy $= r - e$

$= 4 - 2$

$= 2$

The given beam is 2 degree of redundancy.

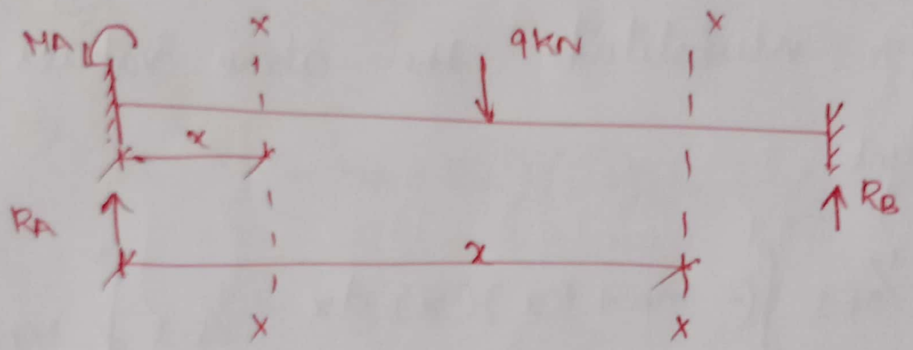
There are four unknowns M_A, M_B, R_A and R_B.

We have only two equations of static

$\Sigma V = 0$

$\Sigma M = 0$

step 2: Let us check M_A and R_A as redundants as shown in the figure.



$$\delta_A = \frac{\partial U_{AB}}{\partial R_A} = 0$$

$$1/EI \int_A^B M_x \frac{\partial M_x}{\partial R_A} dx \quad -① $$

since the end A doesnot rotate we have,

$$\phi_A = \frac{\partial U_{AB}}{\partial M_A} \Rightarrow 1/EI \int_A^B M_x \frac{\partial M_x}{\partial M_A} dx \quad -②$$

Portion	Origin	Limits	EI	M_x	$\frac{\partial M_x}{\partial R_A}$	$\frac{\partial M_x}{\partial M_A}$
AC	A	0 to 1	2EI	$-M_A + R_A x$	x	-1
CB	A	1 to 3	EI	$-M_A + R_A x - 9(x-1)$ $\Rightarrow -M_A + R_A x - 9x + 9$	x	-1

step 4: substitute the above values in eqn ① we get,

$$\Rightarrow \frac{1}{2EI}\int_0^1 (-M_A + Rx)(x)\,dx + \frac{1}{EI}\int_0^3 (-M_A + R_A x - 9 + 9)\,x\,dx = 0$$

$$\Rightarrow \frac{1}{2EI}\int_0^1 (-M_A x + Rx^2)\,dx + \frac{1}{EI}\int_1^3 (-M_A x) + R_A x^2 - 9x^2 + 9x)\,dx = 0$$

$$\Rightarrow \frac{1}{2EI}\left[-M_A - \frac{x^2}{2} + R_A \frac{x^3}{3}\right]_0^1 + \frac{1}{EI}\left[-M_A\frac{x^2}{2} + R_A\frac{x^3}{3} + 9\frac{x^2}{2} + 9\frac{x^3}{3}\right] = 0$$

$$\Rightarrow \frac{1}{2EI}\left[-0.5M_A + 0.33M_A\right] + \frac{1}{EI}\left[-4.5M_A + 9R_A - 81 + 40.5 + 0.5M_A - 0.33R_A + 3 - 4.5\right] = 0$$

$\Rightarrow \frac{1}{EI}\left[-0.25M_A + 0.165R_A - 4M_A + 8.67R_A - 42\right] = 0$

$-4.25\, M_A + 8.835\, R_A = 42$

$\div 4.25 \Rightarrow$ $\quad -M_A + 2.08 R_A = 9.88 \quad — (3)$

substitute the above values in eqn (2) we get,

$$\Rightarrow \frac{1}{2EI}\int_0^1 (-M_A + Rx)(-1)\,dx + \frac{1}{EI}\int_1^3 -M_A + R_A x - 9x + 9\ (-1)\,dx = 0$$

$$\Rightarrow \frac{1}{2EI}\int_0^1 (M_A - Rx)\,dx + \frac{1}{EI}\int_1^3 (M_A - R_A x + 9x - 9)\,dx = 0$$

$$\Rightarrow \frac{1}{2EI}\left[M_A x - \frac{Rx^2}{2}\right]_0^1 + \frac{1}{EI}\left[M_A x - R_A \frac{x^2}{2} + \frac{9x^2}{2} - 9x\right]_1^3 = 0$$

$$\Rightarrow \frac{1}{2EI}\left[M_A - 0.5R_A\right] + \frac{1}{EI}\left[(3M_A + 4.5R_A + 40.5 - 27) - (M_A - 0.5R_A + 4.5 - 9)\right] = 0$$

$$\Rightarrow \frac{1}{EI}\left[0.5M_A - 0.25R_A\right] + \frac{1}{EI}\left[2M_A - 4R_A + 18\right] = 0$$

$$\Rightarrow \frac{1}{EI}\left[0.5M_A - 0.25R_A + 2M_A + 4R_A + 18\right] = 0$$

$\Rightarrow 2.5M_A - 4.25 R_A = -18$

$\div 2.5 \Rightarrow$

$\Rightarrow M_A - 1.7R_A = -7.2 \quad - (4)$

step 5: To find R_A and M_A

By solving the equation (3) & (4) we get,

eqn (3)+(4) we get,

$0.38 R_A = 2.67$

$R_A = 7.05$ kN

Assumed direction is correct

$M_A = 4.79$ kNm

step 6: To find M_B and R_B

Take moment about B,

$\Sigma M_B = 0$

$R_A(3) - M_A - (9 \times 2) + M_B = 0$

$3(7.05) - 4.39 - 18 + M_B = 0$

$M_B = 1.64$ kNm

$\Sigma V_B = 0$

$R_A + R_B = 9$

$R_B = 9 - 7.05$

$R_B = 1.95\ kNm$

CE8502 - STRUCTURAL ANALYSIS - I

INTROBUTION :

Structural Analysis :-

Structural analysis is the determination of the effects of loads on physical structures and their components.

Structure :

A structure can be a column, a beam, a floor, a slab and arch, a truss or whatever, so when we are analysing a structure we applied the basic Principles of mechanics and for any structure geometric and applied forces we produce to determine

⇒ Relative forces
⇒ Internal forces
⇒ Deformations
⇒ Deflections etc.,

with the aim of determining, if the structure is safe or not.

Strength and Stability :-

→ The structure should be strong and stable.

→ A structure must be both strong and stable. If it is not strong it will break and collapse (snap (or) crack (or) crush). This can happen when the material not strong enough to take the stress comming on it.

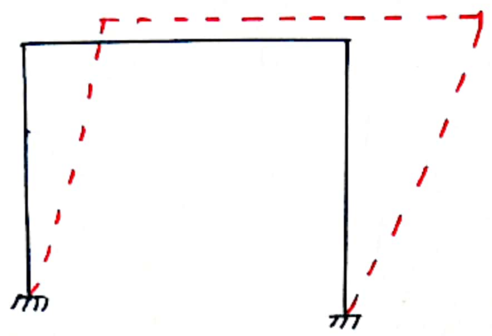
→ If a structure is not stable it will move (or) fall (or) slide even in the material of the component Part has not broken (fail)

→ If a structure unstable the following may occur,

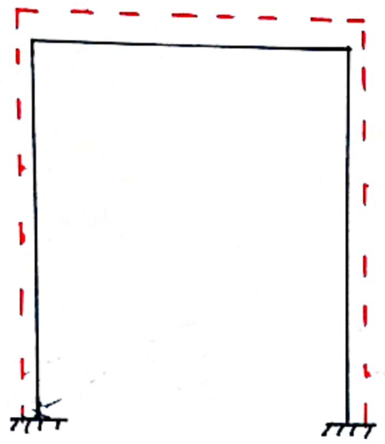
1) sliding
2) Overturning
3) Sinking
4) Rotating about horizontal (or) vertical axis.

Serviceability :

The structure is analysed, designed and constructed in order to be of use. (In order to be serviceability).



Weak structure (Break)



Unstable structure (Displaces)

Statically Determine structure :

External and internal forces that can keep the structures in equlibrium under applied loads or unique. such structures are called statically determinate or simply determinate structures.

Equlibrium Conditions:

$$\Sigma H = 0$$
$$\Sigma V = 0$$
$$\Sigma M = 0$$

Example :-

* Simply suppoted beam
* Cantilever beam
* Overhanging

Statically indeterminate structure :-

The structures when we provide more supports that the minimum required for stability. This are also called hyperstatic structure (or) redundant structure, since there is a redundancy.

Example :-

Fixed beam
Continuous beam
Rigid Frames
Pin Joint.

Internal redundancy
Members

External redundancy
Too many supports

Compatability :

It is similar to concept to equlibrium forces should be in equlibrium.

Displacement of structure should be compatible.

Type of analysis:-

⇒ Stiffness Method

i) Moment distribution method.
ii) Slope deflection method.
iii) Stiffness matrix method.
iv) Kani's Method.

⇒ Flexibility Method

i) Energy method
ii) Flexibility matrix method
iii) Column analogy method

Determinate Structure	Indeterminate Structure
It can be analysed from conditions of static alone. so easy to analyse.	It cannot be analysed from condition of static condition. We would need in additional compatibility of displacement condition. Analysed required more effort
Reactive forces are independent of member Properties like E, A, I etc.,	Dependent on member properties.
Temperature changes do not create additional forces / stresses.	Temperature changes do create additional forces / stresses.
There would be no forces due to lack of fit.	Lack of fit would produce internal forces.
They do not have any sectional, second line of defence. If one of the member fail the structure would collapse.	For every additional redundancy. There is some additional safety against collapse.

UNIT-II

Slope Deflection Method

→ This method was first proposed by Prof. GEORGE A. MANEY in 1915

→ It is ideally suited to the analysis of continuous beams and rigid jointed frames.

→ Basic unknowns like slopes and deflection of joints are found out.

→ The development of this method is the matrix forms is "stiffness matrix method".

→ It is commonly used for the analyses of large structures with the help of computers.

Assumptions:

→ All joints are rigid.

→ The rotations of joints are treated as unknowns.

→ Between each point of the support the beam section is constant.

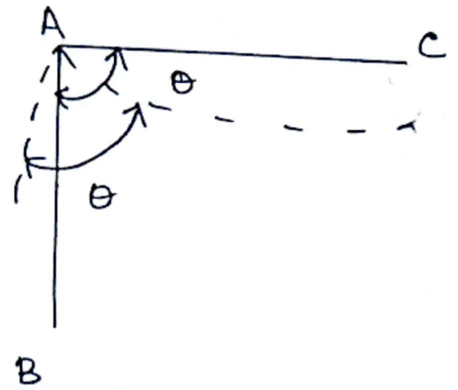
→ The joint in structure may rotate (or) deflect as a whole, but the angles between the members meeting at the joint remain the same.

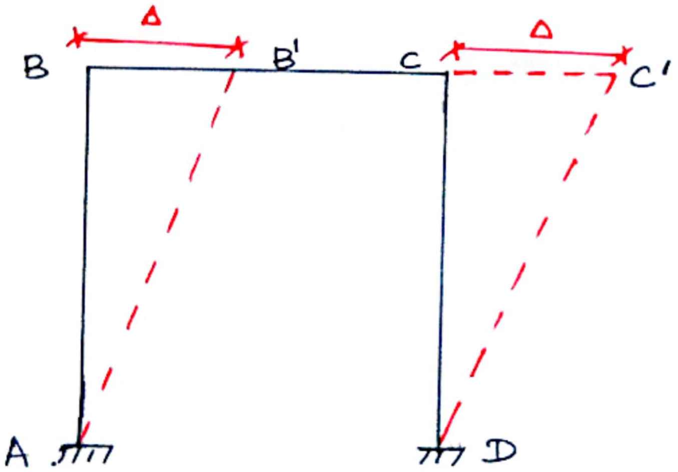
→ Distances due to axial deformations are neglected.

→ Shear deformation are neglected.

→ Axial, shear (small) → Neglected.

→ Flexure → Countable.

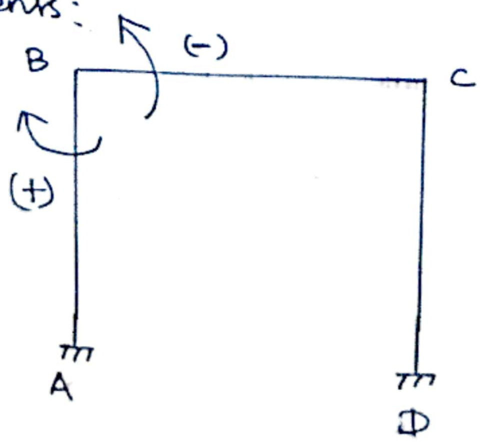




BB' = CC' = Δ

sign conventions:

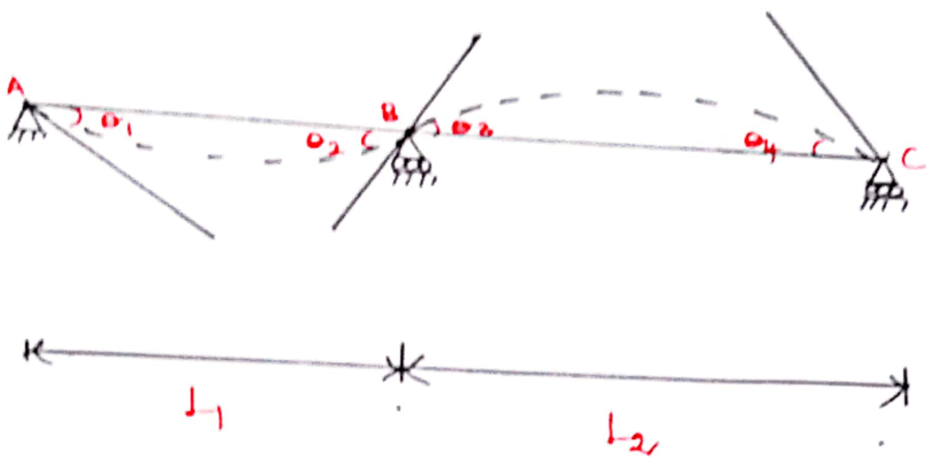
Moments:



clockwise moments (↻) = +ve

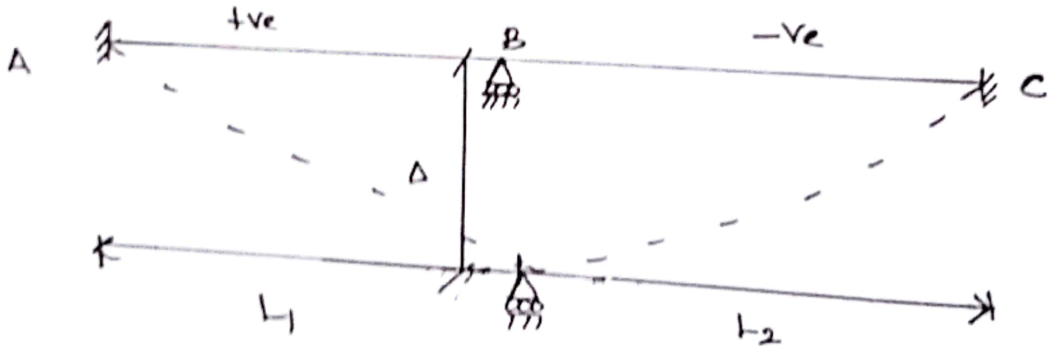
Anticlockwise moments (↺) = −ve

Rotations :



Clockwise rotations (↻) = +ve
Anticlockwise rotations (↺) = −ve

Settlements :-



Right side support in below the left side support = (−) ve

Left side support in below the Right side support = (+) ve

Application of slope deflection equations:-

Rigid jointed structure can be analysed.
Analysis of continuous beams.
Frames without sway (Non- sway)
Frames with side sway (sway)

Limitations.

It is not easy to account for varying member sections.

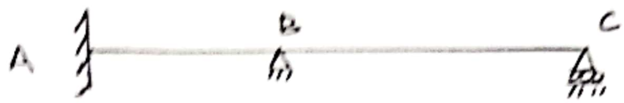
It becomes very cambersome when the unknown displacements are large in number.

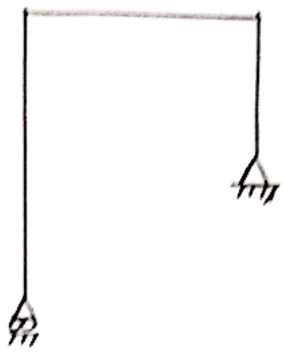
Why slope deflection method is called displacement method?

In slope deflection method, displacements (like slope and displacements) are treated as unknowns and hence the method is called as a displacement method.

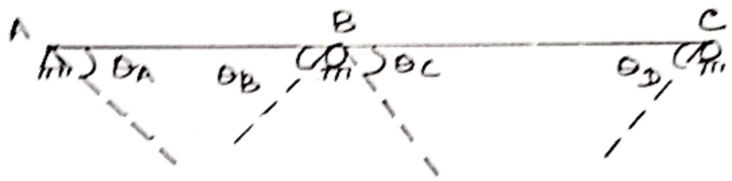
Degree of Freedom:

In a structure the degree of the number of independent joint displacements that the structure can undergoes are known as degree of Freedom.





Slope deflection Equation:



$$M_{AB} = M_{FAB} + \frac{4EI\theta_A}{l} + \frac{2EI\theta_B}{l} \pm \frac{6EI\delta}{l^2}$$

$$M_{AB} = M_{FAB} + \frac{2EI}{l}\left[2\theta_A + \theta_B \pm 3\delta/l\right]$$

$$M_{AB} = M_{FAB} + \frac{2EI}{l}\left[2\theta_A + \theta_B\right] \quad (\because \delta = 0)$$

similarly at support 'B':

$$M_{BA} = M_{FBA} + \frac{2EI\theta_A}{l} + \frac{4EI\theta_B}{l} \pm \frac{6EI\delta}{l^2}$$

$$M_{BA} = M_{FBA} + \frac{2EI}{l}\left[\theta_A + 2\theta_B\right] \quad (\because \delta = 0)$$

Unknowns : θ_A, θ_B, θ_C

Equilibrium equations:-

$$M_{AB} = 0$$

$$M_{CB} = 0$$

$$M_{BA} + M_{BC} = 0$$

M_{AB} = Final moment for span AB

M_{FAB} = Fixed end moment

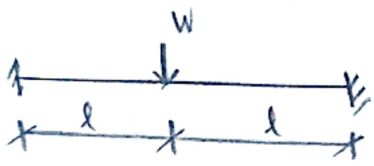
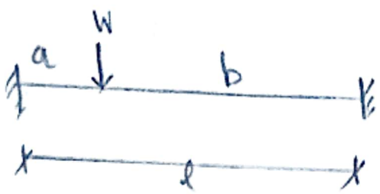
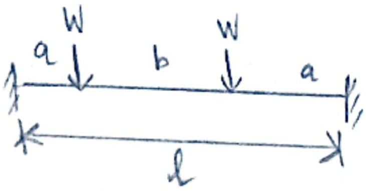
θ_A = slope at joint "A"

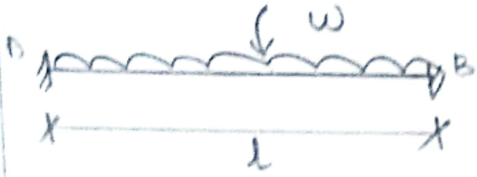
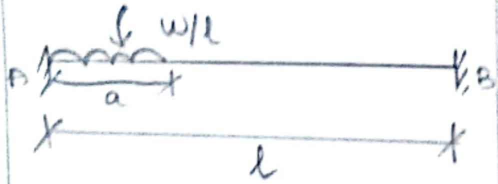
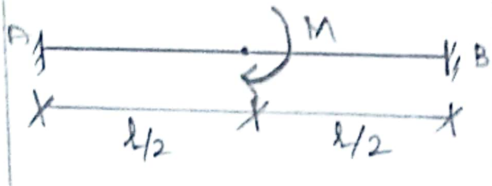
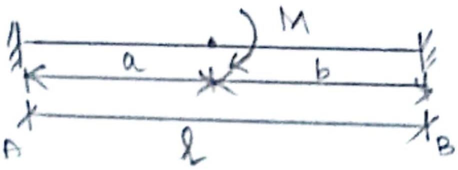
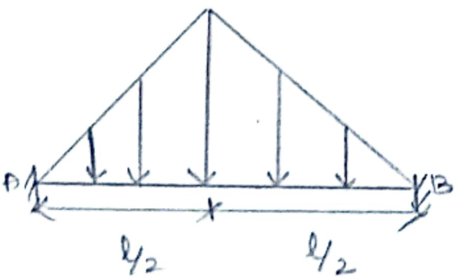
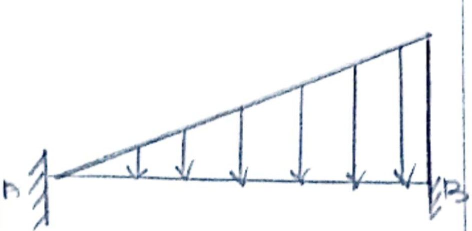
θ_B = slope at joint "B"

l = Length of the member

E = Youngs Modulus

I = Moment of inertia

Cases	M_{FAB}	M_{FBA}
1. 	$-\frac{wl}{8}$	$+\frac{wl}{8}$
2. 	$-\frac{wab^2}{l^2}$	$+\frac{wa^2b}{l^2}$
3. 	$-\frac{wa(a+b)}{l}$	$+\frac{wa(a+b)}{l}$

Cases	M_{FAB}	M_{FBA}
4. 	$-\frac{wl^2}{12}$	$+\frac{wl^2}{12}$
5. 	$\frac{-wa^2(bl^2-8ab+3a^2)}{12l^2}$	$\frac{wa^3(4l-3a)}{12l^2}$
6. 	$M/4$	$M/4$
7. 	$\frac{Mb}{l^2}(3a-b)$	$\frac{Ma}{l^2}(3b-l)$
8. 	$\frac{-5wl^2}{96}$	$\frac{+5wl^2}{96}$
9. 	$\frac{-wl^2}{30}$	$\frac{+wl^2}{20}$

1

Procedure :

1. To find fixed end moments for the given loads (FEM).

2. To find slope deflection equations for each span

3. Applying equilibrium conditions

4. To find final moments

5. Check with equilibrium conditions

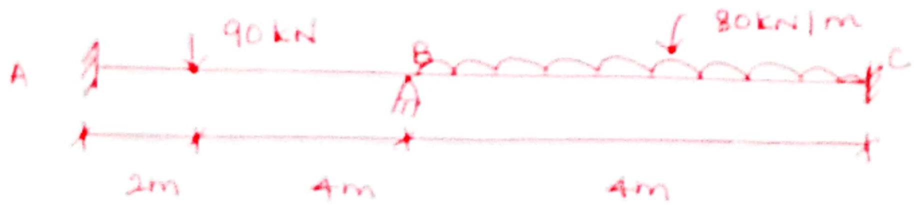
6. To find free BMD

7. To find reactions

Analysis of continuous beams by slope deflection method (Without settlement or sinking of supports)

Problem 1 :

Analyse the beam showing in the figure by slope deflection method .



soln:-

step 1: To Find fixed end Moments

For span AB,

$$M_{FAB} = -\frac{Wab^2}{l^2}$$

here, $a = 2m$, $b = 4m$, $l = 6m$

$$= -90 \times 2 \times 4^2 / 6^2$$

$$M_{FAB} = -80 kNm$$

$$M_{FBA} = \frac{Wa^2b}{l^2}$$

$$= \frac{90 \times 2^2 \times 4}{6^2}$$

$$M_{FBA} = +40 kNm$$

For span BC,

$$M_{FBC} = -wL^2/12$$

here, $L = 4m$

$$= -\frac{80 \times 4^2}{12}$$

$$= -106.67 kNm$$

$$M_{FCB} = \frac{wl^2}{12}$$

$$= \frac{80 \times 4^2}{12}$$

$$= 106.67 kNm$$

step 2 : To find slope deflection equation

For span AB,

$$M_{AB} = M_{FAB} + \frac{2EI}{l}\left[2\theta_A + \theta_B + 3\delta/l\right]$$

Here, No settlement ($\because \delta = 0$)

$$M_{AB} = M_{FAB} + 2EI/l\left[2\theta_A + \theta_B\right]$$

$$= -80 + \frac{2EI}{(4+2)}(\theta_B)$$

$$M_{AB} = -80 + 0.33\,EI\,\theta_B \quad \text{——— (1)}$$

$$M_{BA} = M_{FBA} + \frac{2EI}{l}\left[\theta_A + 2\theta_B\right]$$

$$= 40 + \frac{2EI}{l}\left[0 + 2(\theta_B)\right]$$

$$M_{BA} = 40 + 0.67\,EI\,\theta_B \quad \text{——— (2)}$$

For span BC,

$$M_{BC} = M_{FBC} + 2EI/l\left[2\theta_B + \theta_C\right]$$

$$= -106.67 + 2EI/4\,(2\theta_B)$$

$$M_{BC} = -106.67 + EI\,\theta_B \quad \text{——— (3)}$$

$$M_{CB} = M_{FCB} + 2EI/l\,(\theta_B + 2\theta_C)$$

$$= 106.67 + 2EI/l\,(\theta_B)$$

$$M_{CB} = 106.67 + 0.5\,EI\,\theta_B \quad \text{——— (4)}$$

Step 3 : Apply equlibrium conditions

At joint B, moment about BA and moment about AB = 0.

$$M_{BA} + M_{BC} = 0$$

$$40 + 0.67\,EI\theta_B - 106.67 + EI\theta_B = 0.$$

$$-66.67 + 1.67\,EI\theta_B = 0.$$

$$1.67\,EI\theta_B = 66.67$$

$$EI\theta_B = 39.92$$

Step 4 : To find Final Moments :

① ⇒ $M_{AB} = -80 + 0.33 \times 39.92 = -66.83$ kNm

② ⇒ $M_{BA} = 66.75$ kNm

③ ⇒ $M_{BC} = -66.75$ kNm

④ ⇒ $M_{CB} = 126.63$ kNm.

Step 5 : Check for equlibrium conditions :-

$$M_{BA} + M_{BC} = 0$$

$$66.75 - 66.75 = 0.$$

Step 6 : Free Bending moment Diagram :

For span AB,

$$Wab/l = 90 \times 2 \times 4/6 = 120 \text{ kNm}.$$

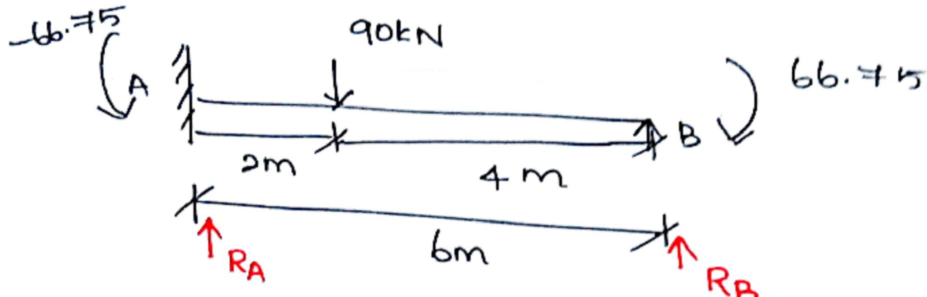
Scanned with OKEN Scanner

For span BC,

$$\frac{Wl^2}{8} = \frac{80 \times 4^2}{8} = 160 \text{ kNm}$$

Step 7 : To find Reactions :

Span AB:



$$(R_A \times 6) - (90 \times 4) - 66.75 + 66.75 = 0$$

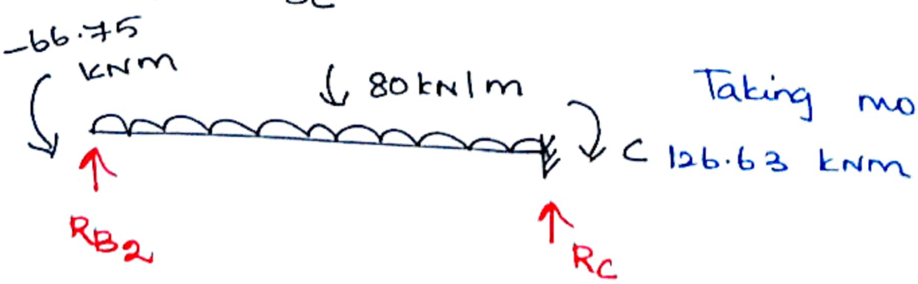
$$R_A = 60 \text{kN}$$

$$R_A + R_B = 90$$

$$60 + R_B = 90$$

$$R_B = 30 \text{ kN}$$

For span BC



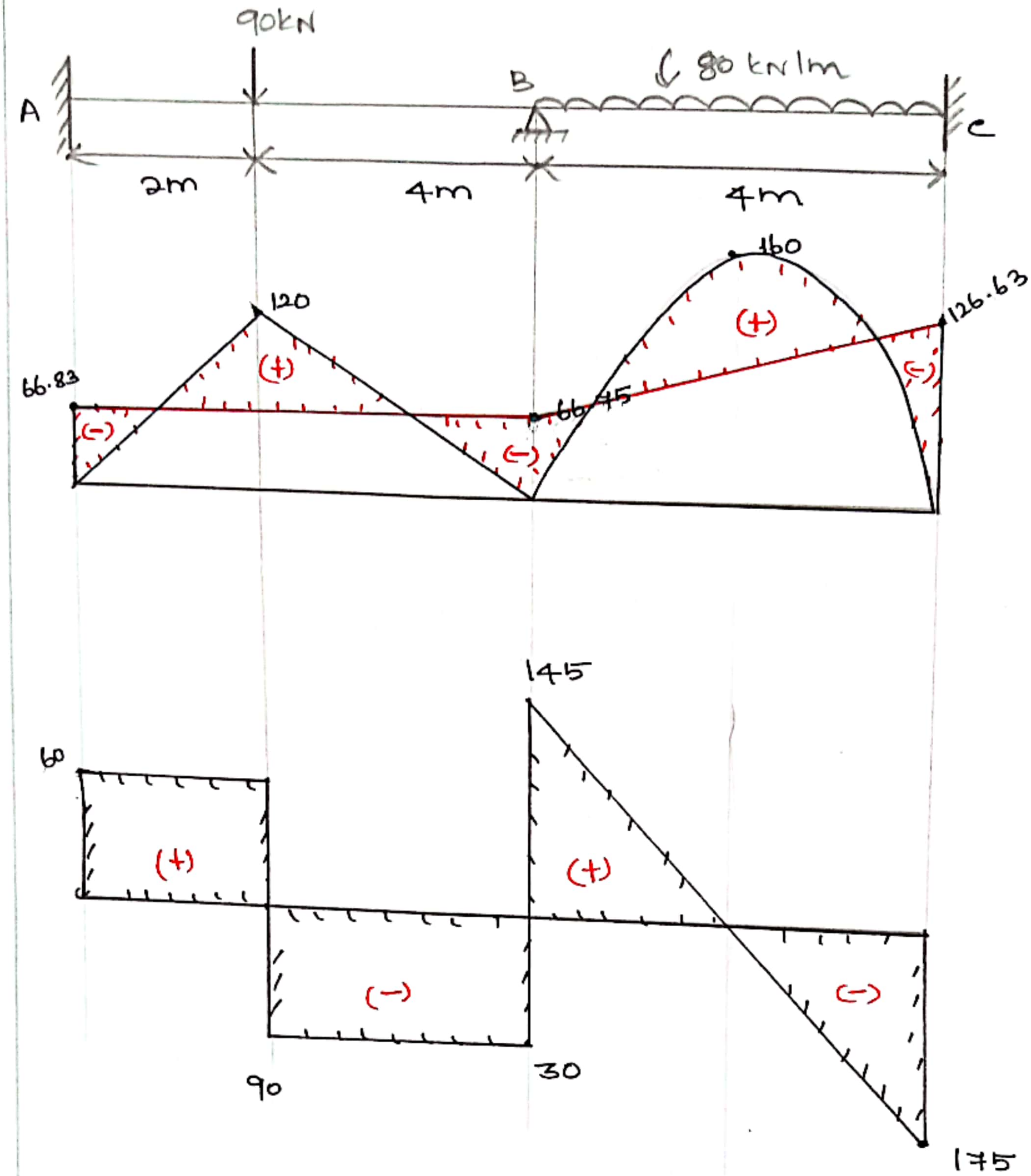
Taking moment about "c"

$$(R_{B2} \times 4) - \left(80 \times 4 \times \tfrac{4}{2}\right) - 66.75 + 126.63 = 0$$

$$R_{B2} = 145 \text{ kN}$$

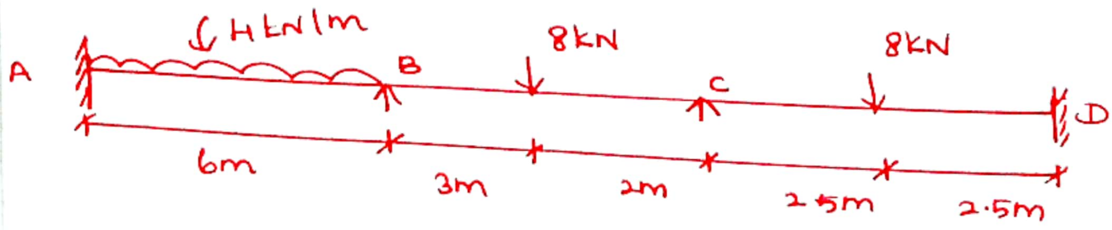
$$R_{B2} + R_C = 80 \times 4$$

$$R_C = 175 \text{ kN}$$



Problem 2:

A continuous beam ABCD consists of 3 span and is loaded as shown in figure. Analyse a beam by using slope deflection method "E" is constant throughout.



step 1 : To Find Fixed End Moments,

$$M_{FAB} = -\frac{wl^2}{12} = -\frac{4\times 6^2}{12} = -12 \text{ kN m}$$

$$M_{FBA} = \frac{wl^2}{12} = \frac{4\times 6^2}{12} = 12 \text{ kN/m}$$

$$M_{FBC} = -\frac{wab^2}{l^2} = -\frac{8\times 3\times 2^2}{5^2} = -3.84 \text{ kNm}$$

$$M_{FCB} = +\frac{wa^2b}{l^2} = \frac{8\times 3^2\times 2}{5^2} = 5.76 \text{ kN/m}$$

$$M_{FCB} = -\frac{wl}{8} = -\frac{8\times 5}{8} = -5 \text{ kN/m}$$

$$M_{FBC} = \frac{wl}{8} = \frac{5\times 8}{8} = 5 \text{ kN/m}$$

step 2 : To find slope deflection equation :

$$M_{AB} = M_{FAB} + \frac{2EI}{l}(2\theta_A + \theta_B)$$

$$M_{AB} = -12 + EI\theta_B \quad —— ①$$

$$M_{BA} = M_{FBA} + \frac{2EI}{l}(2\theta_B + \theta_A)$$

$$M_{BA} = 12 + 2EI\theta_B \quad —— ②$$

$$M_{BC} = M_{FBC} + \frac{2E}{l}I(2\theta_B + \theta_C)$$

$$M_{BC} = -3.84 + \frac{2E(2I)}{l}(2\theta_B + \theta_C)$$

$$M_{BC} = -3.84 - 1.6EI\theta_B + 0.8EI\theta_C \quad — ③$$

$$M_{CB} = M_{FCB} + \frac{2EI}{l}(2\theta_C + \theta_B)$$

$$= 5.76 + 1.6EI\theta_C + 0.8EI\theta_C \quad \text{— (4)}$$

$$M_{CD} = M_{FCD} + \frac{2EI}{l}(2\theta_C + \theta_D)$$

$$M_{CD} = -5 + 1.6EI\theta_C \quad \text{— (5)}$$

$$M_{DC} = M_{FDC} + \frac{2EI}{l}(2\theta_D + \theta_C)$$

$$M_{DC} = 5 + 0.8EI\theta_C \quad \text{— (6)}$$

step 3: Equlibrium conditions:

At joint B,

$$M_{BA} + M_{BC} = 0$$

$$12 + 2EI\theta_B + 1.6EI\theta_B + 0.8EI\theta_C - 3.84 = 0$$

$$3.6EI\theta_B + 0.8EI\theta_C = -8.16 \quad \text{— (7)}$$

At joint C,

$$M_{CB} + M_{CD} = 0$$

$$5.76 + 1.6EI\theta_C + 0.8EI\theta_B - 5 + 1.6EI\theta_C = 0$$

$$3.2EI\theta_C + 0.8EI\theta_B = -0.76 \quad \text{— (8)}$$

solving (7) & (8),

$$EI\theta_B = -2.34$$

$$EI\theta_C = 0.348$$

Step 4: Finding Final moments:

$$M_{AB} = -12 + (-2.34) = -14.34 \text{ kNm}$$

$$M_{BA} = 12 + (-2.34) \times 2 = 7.32 \text{ kNm}$$

$$M_{BC} = -7.32 \text{ kNm}$$

$$M_{CB} = +4.43 \text{ kNm}$$

$$M_{CD} = -4.43 \text{ kNm}$$

$$M_{DC} = 5.28 \text{ kNm}$$

Step 5: Check for equlibrium conditions:

at joint B,

$$M_{BA} + M_{BC} = -7.32 + 7.32 = 0$$

At joint C,

$$M_{CB} + M_{CD} = 4.43 - 4.43$$
$$= 0$$

Hence Ok.

Step 6: Find bending moment:

For span AB,

$$wl^2/8 = \frac{4 \times 6^2}{8} = 18 \text{ kNm}$$

For span BC,

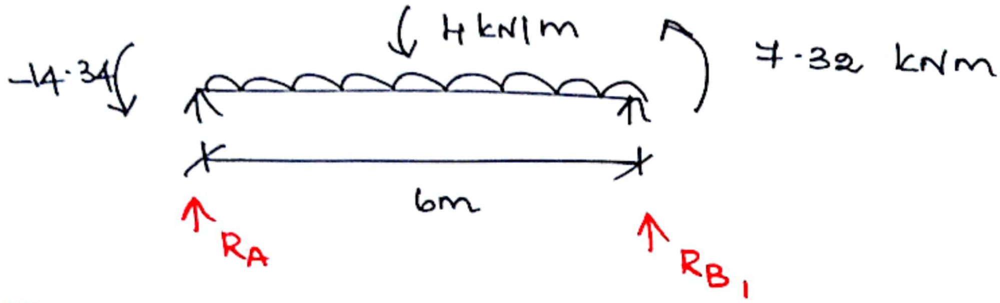
$$Wab/l = \frac{8 \times 3 \times 2}{5} = 9.6 \text{ kNm}$$

For span CD,

$$\frac{wl}{4} = \frac{8 \times 5}{4} = 10 \text{ kNm}.$$

Step 7: To find Reactions:

For span AB,



Taking moment about B,

$$(R_A \times 6) - (4 \times 6 \times 6/2) + 7.32 - 14.34 = 0$$

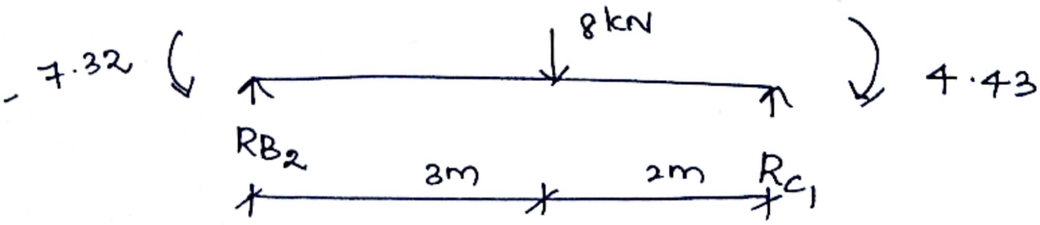
$$6R_A = 79.02$$

$$R_A = 13.17 \text{ kN}$$

$$R_A + R_{B_1} = 24.$$

$$R_{B_1} = 10.83 \text{ kN}$$

For span BC,



Taking moment about C,

$$(R_{B2} \times 5) - (8 \times 2) + 4.43 - 7.32 = 0$$

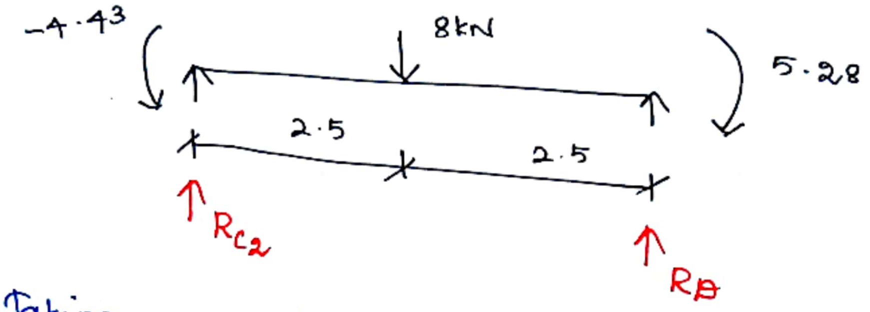
$5R_{B_2} = 18.89$

$R_{B_2} = 3.78\,kN$

$R_{B_2} + R_{C_1} = 8\,kN$

$R_{C_1} = 4.22\,kN$

For span CD,



Taking moment about R_D,

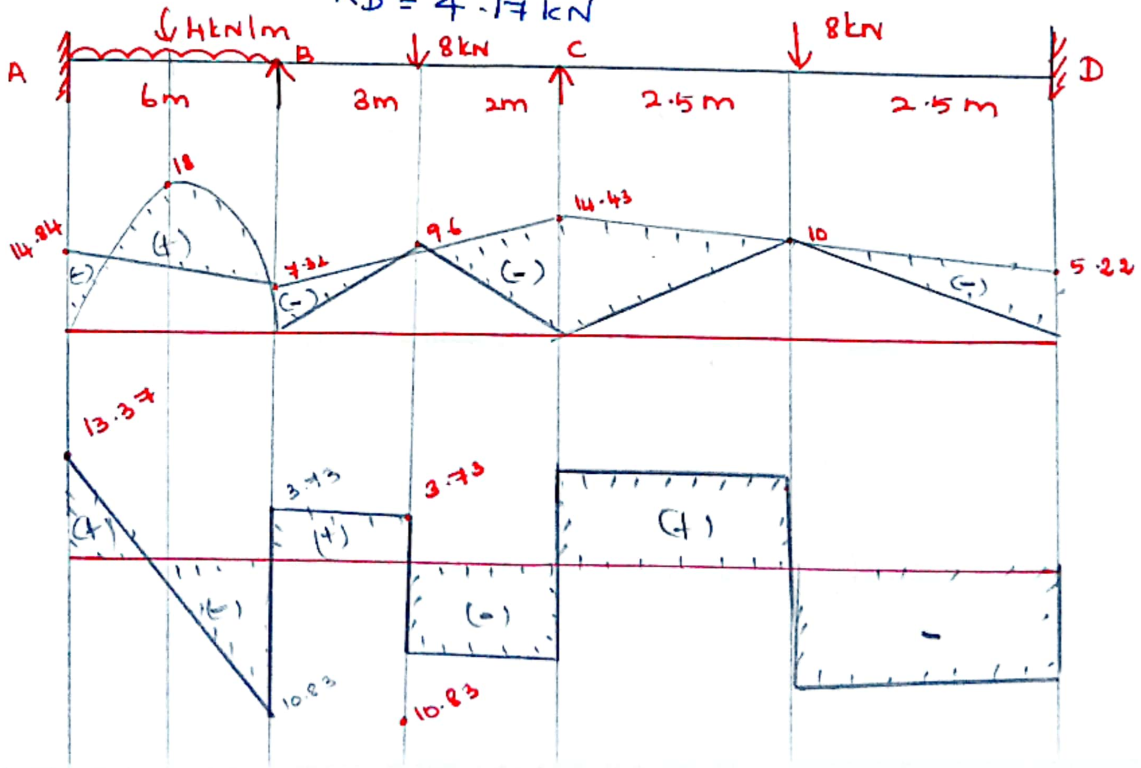
$(R_{C_2} \times 5) - 8 \times 2.5 - 4.43 + 5.28 = 0$

$5R_{C_2} = 19.15$

$R_{C_2} = 3.83$

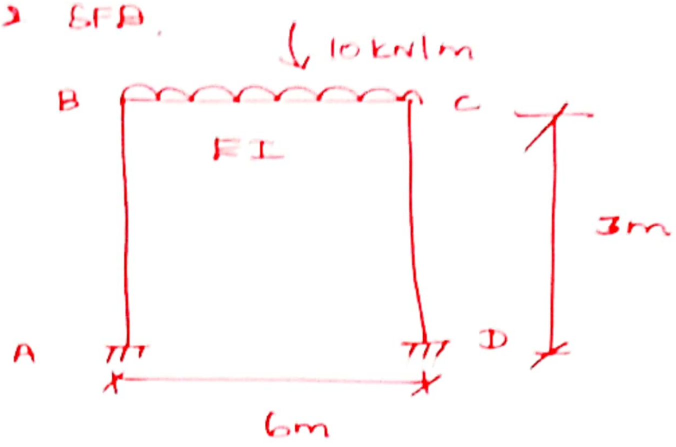
$R_{C_2} + R_D = 8$

$R_D = 4.17\,kN$



Analysis of Portal Frames (Non - sway)

Analyse the portal frame shown in the figure by slope deflection method and sketch the BMD & SFD.



Soln:

Step 1: Find Fixed end moments:-

$M_{FAB} = 0$

$M_{FBA} = 0$

$M_{FBC} = -WL^2/12$

$= -10 \times 6^2/12 = -30$ kNm

$M_{FCB} = WL^2/12$

$= 10 \times 6^2/12 = 30$ kNm.

$M_{FCD} = 0$

$M_{FDC} = 0$

Step 2: slope deflection equation:

For span AB,

$l = 3m$; $\theta_A = 0$;

$$M_{AB} = M_{FAB} + \frac{2EI}{l}\left[2\theta_A + \theta_B\right]$$

$$M_{AB} = \frac{2EI\theta_B}{3} + (0) = 0.67EI\theta_B$$

$$M_{BA} = M_{FBA} + 2EI/l \left[\theta_A + 2\theta_B\right]$$

$$M_{BA} = 0 + \frac{2EI}{3}\left[2\theta_B\right] = 1.33EI\theta_B$$

For span BC,

$l = 6m$.

$$M_{BC} = M_{FBC} + 2EI/l\,(2\theta_B + \theta_C)$$

$$= -30 + \frac{2EI}{6}(2\theta_B + \theta_C)$$

$$M_{BC} = -30 + 0.67EI\theta_B + 0.33EI\theta_C$$

$$M_{CB} = M_{FCB} + 2EI/l\,(\theta_B + 2\theta_C)$$

$$M_{CB} = 30 + 0.33EI\theta_B + 0.67EI\theta_C$$

For span CD,

$l = 3m$,

$$M_{CD} = M_{FCD} + \frac{2EI}{3}(2\theta_C + \theta_D)$$

$$M_{CD} = 1.33EI\theta_C$$

$$M_{DC} = M_{FDC} + \frac{2EI}{3}(\theta_C + 2\theta_D)$$

$$M_{DC} = 0.67 EI\theta_C$$

step 3 : Find Equlibrium conditions.

$\Sigma M_B = 0$; $M_{BA} + M_{BC} = 0$

$\Sigma M_C = 0$; $M_{BC} + M_{CD} = 0$

Applying Equlibrium conditions,

$$M_{BA} + M_{BC} = 0$$

$$1.33EI\theta_B + 0.67\,EI\theta_B + 0.33EI\theta_C = 0$$

$$2EI\theta_B + 0.33\,EI\theta_C = 30 \quad \text{——} \; ①$$

$$M_{CB} + N_{CB} = 0$$

$$0.33EI\theta_B + 0.67EI\theta_C + 1.33\,EI\theta_C = -30$$

$$0.33EI\theta_B + 2EI\theta_C = -30 \quad \text{——} \; ②$$

① & ② =>

$$EI\theta_B = 17.96$$

$$EI\theta_C = -17.96$$

step 4 : Final Moments :

$M_{AB} = 0.67\,EI\theta_B = 12.03$ kNm

$M_{BA} = 23.89$ kNm

$M_{BC} = -23.89$ kNm

$M_{CB} = 23.89$ kNm

$M_{CB} = -23.89$ kNm

$M_{BC} = -12.63$ kNm

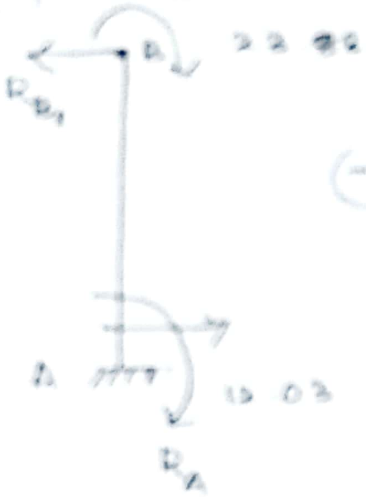
step 5: check for equilibrium conditions

$M_{BA} + M_{BC} = 0$

$M_{CD} + M_{DC} = 0$

step 6: To find support reactions

span AB

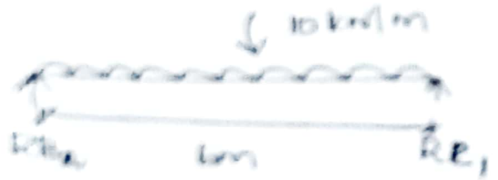


$(-R_A \times 3) + 12.03 + 23.88 = 0$

$R_A = 12$

$R_{B_1} = -12$

span BC



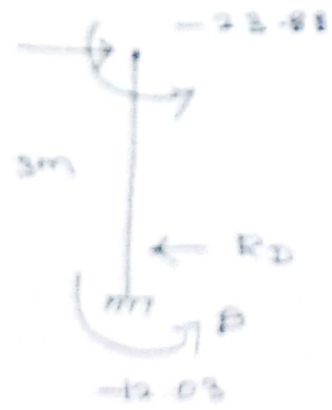
$(R_{B_2} \times 6) - (10 \times 6 \times 6/2) - 23.88 + 23.88 = 0$

$R_{B_2} = 30 kN$

$R_{B_2} + R_{C_1} = 10 \times 6$

$R_{C_1} = 30$

span CD



$(R_D \times 3) - 12.03 - 23.88 = 0$

$R_D = 12 kN$

$R_{C_2} = -12 kN$

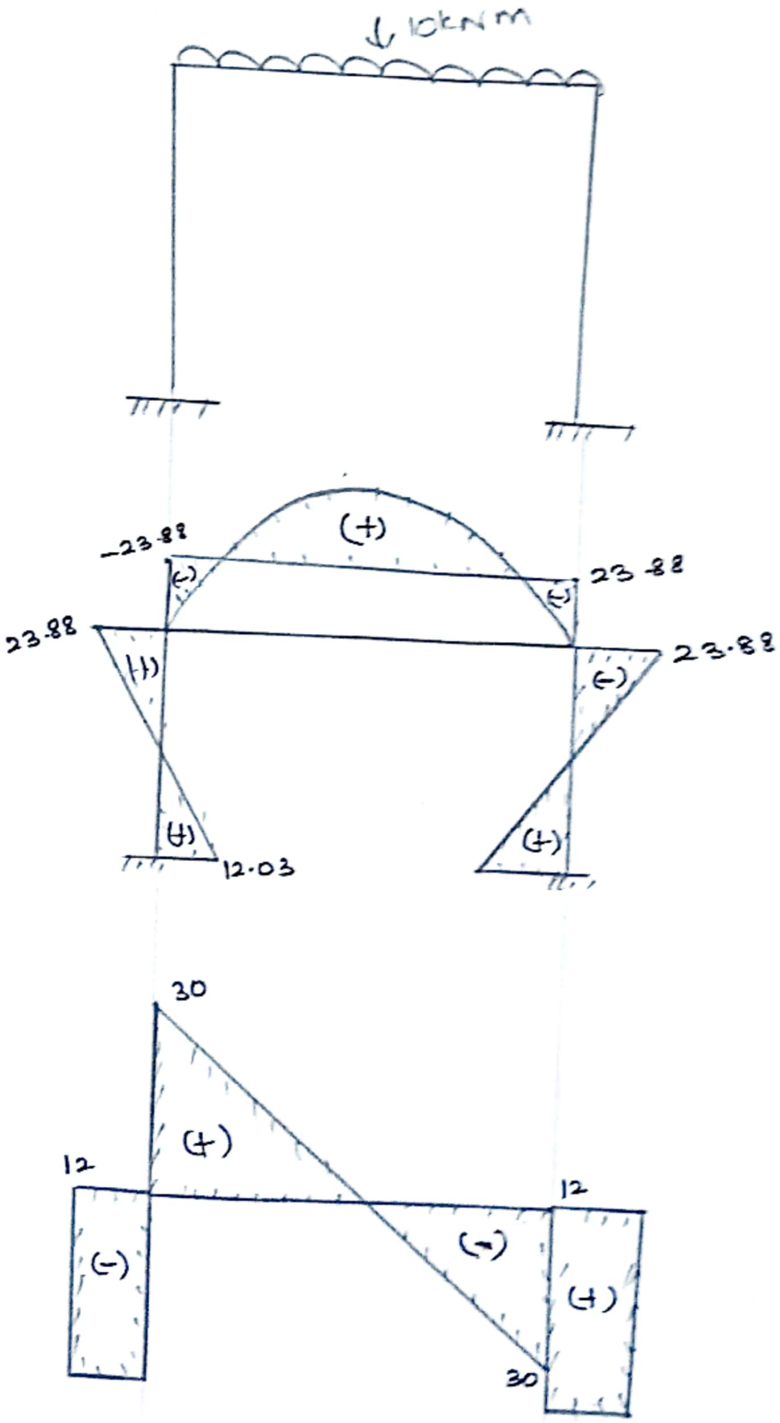
Step 7 : Free bending Moment :

Span $AB = 0$

Span $BC = wL^2/8$

$$= \frac{10 \times 6^2}{8} = 45 \text{ kN/m}$$

Span $CD = 0$



Analyses of portal Frames with side sway

When the horizontal forces are applied to the frames it will sway some joints move unknown distances. Portal frames may sway due to one of the following reasons:

Eccentric or unsymmetrical loading on the portal frames.

Unsymmetrical loading on the portal frames unsymmetrical shape of the frames.

Different end conditions of the columns of the portal frame.

Non-uniform section of the members of the frame.

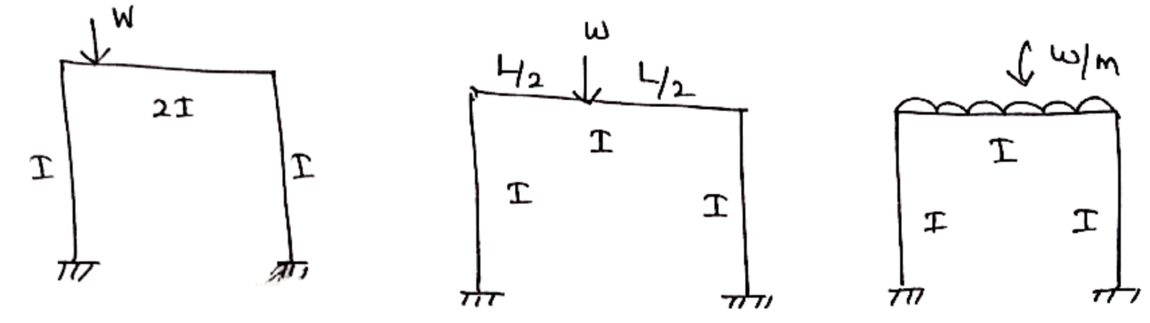
Horizontal loading on the columns of the frame.

Settlement of the support of the frame.

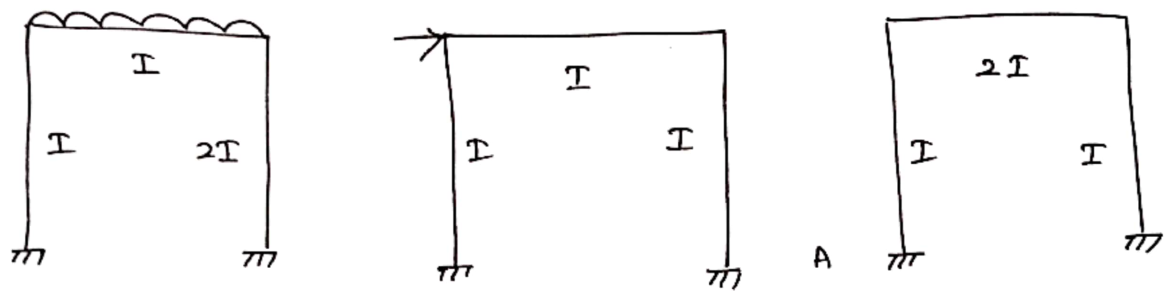
A combination of the above.

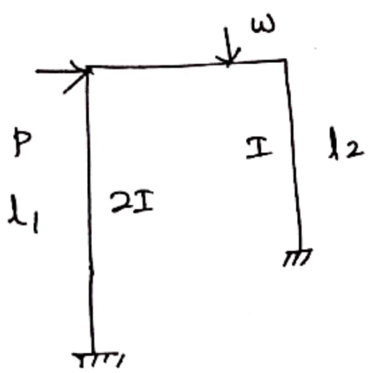
The amount of joint moment or sway is not known and forms an additional unknown.

In slope deflection method, to evaluate the additional unknown shear equation is made use of.

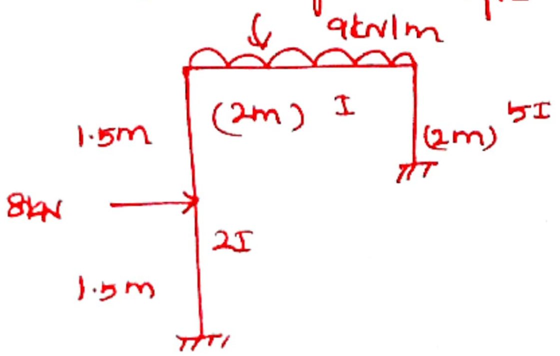


(i) (ii) (iii)





Analyse completely the portal frame shown in the figure by slope deflection method.



soln:

step 1: To find fixed end moments:

$$M_{FAB} = -\frac{Wl}{8} = \frac{-8\times 3}{8} = -3 \text{ kNm}$$

$$M_{FBA} = \frac{Wl}{8} = \frac{8\times 3}{8} = 3 \text{ kNm}$$

$$M_{FBC} = -\frac{WL^2}{12} = -\frac{9\times 2^2}{12} = -3 \text{ kNm}$$

$$M_{FCB} = \frac{WL^2}{12} = \frac{9\times 2^2}{12} = 3 \text{ kNm}$$

$$M_{FCD} = 0$$

$$M_{FDC} = 0$$

step 2: To find slope deflection equations:

For span AB,

$l = 3m$, $EI = 2EI$, $\theta_A = 0$; $\theta_B = ?$

[Right side sway always Negative]

$$M_{FAB} = M_{FAB} + \frac{2EI}{l}\left[2\theta_A + \theta_B + 3\delta/l\right]$$

$$= -3 + \frac{2(2EI)}{3}\left[\theta_B \pm 3\delta/3\right]$$

$$= -3 + 1.33\, EI\theta_B \pm 1.33\, EI\delta$$

$$M_{BA} = M_{FBA} + \frac{2(2EI)}{l}\left[\theta_A + 2\theta_B \pm 3\delta/L\right]$$

$$= 3 + 2.67\, EI\theta_B \pm 1.33\, EI\delta$$

For span BC,

$l = 2m$; $\theta_B = ?$ $\theta_C = ?$

$$M_{BC} = M_{FBC} + \frac{2EI}{l}\left[2\theta_B + \theta_C \pm \frac{3\delta}{l}\right]$$

$$= -3 + 2\frac{EI}{2}(2\theta_B + \theta_C)$$

$$= -3 + 2EI\theta_B + EI\theta_C$$

$$M_{BC} = 3 + 2EI\theta_C + EI\theta_B$$

$$M_{CD} = M_{FCD} + \frac{2EI}{l}\left[2\theta_C + \theta_D - 3\delta/l\right]$$

$$= \frac{1.5(2EI)}{2}\left[2\theta_C + \theta_D - 3\delta/2\right]$$

$$= 3EI\theta_C - 2.25EI\delta$$

$$M_{DC} = M_{FCD} + \frac{2 \times 1.5EI}{2}\left[\theta_C + 2\theta_D - 3\delta/l\right]$$

$$= 1.5EI\theta_C - 2.25EI\delta$$

$$M_{DC} = 1.5EI\theta_C - 2.25EI\delta$$

Step 8: Equlibrium conditions:

At joint B,

$$M_{BA} + M_{BC} = 0$$

At joint C,

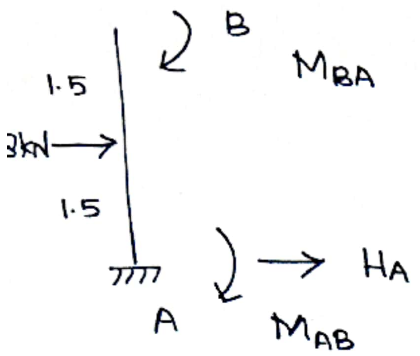
$$M_{CB} + M_{CD} = 0$$

Shear equations,

$$\Sigma H = 0$$

$H_A + H_D = -8$

Span AB:-



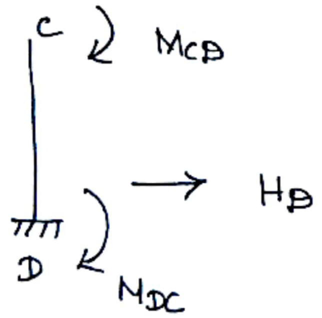
Moments always considered at positive.

Taking moment about B:-

$$M_{AB} + M_{BA} - 12 = 3H_A$$

$$H_A = \frac{M_{AB} + M_{BA}}{3} - 4$$

For span CD:



Taking moment about "C":-

$$M_{CD} + M_{DC} - H_D \times 2 = 0$$

$$H_D = \frac{M_{CD} + M_{DC}}{2}$$

$\Sigma H = 0$;

$$H_A + H_D + 8 = 0$$

$$\frac{M_{AB}}{3} + \frac{M_{BA}}{3} - 4 + \frac{M_{CD}}{2} + \frac{M_{DC}}{2} + 8 = 0$$

$$2M_{AB} + 2M_{BA} + 3M_{CD} + 3M_{DC} = -4 \times 6$$

$$2M_{AB} + 2M_{BA} + 3M_{CD} + 3M_{DC} = -24$$

Applying equilibrium conditions:

$$M_{BA} + M_{BC} = 0$$

$$3 + 2.67\,EI\theta_B - 1.33\,EI\delta - 3 + 2EI\theta_B + EI\theta_C = 0$$

$$4.67\,EI\theta_B + EI\theta_C - 1.33\,EI\delta = 0$$

$$M_{CB} + M_{CD} = 0$$

$$3 + EI\theta_B + 2EI\theta_C + 3EI\theta_C - 2.25\,EI\delta = 0$$

$$EI\theta_B + 5EI\theta_C - 2.25\,EI\delta = -3$$

$$2M_{AB} + 2M_{BA} + 3M_{CD} + 3M_{DC} = -24$$

$$2(-3 + 1.33EI\theta_B - 1.33EI\delta) + 2(3 + 6.27EI\theta_B - 1.33EI\delta) + 3(3EI\theta_C - 2.25\,EI\delta) + 3(1.5EI\theta_C - 2.25\,EI\delta) = -24$$

$$-6 + 2.66EI\theta_B - 2.66\,EI\delta + 6 + 5.34\,EI\theta_B - 2.66\,EI\delta + 9EI\theta_C - 6.75EI\delta + 4.5\,EI\theta_C - 6.75\,EI\,\delta = -24$$

$$8EI\theta_B + 13.5\,EI\theta_C - 18.82EI\delta = -24$$

solving equations :-

$$EI\theta_B = 0.41$$

$$EI\theta_C = -0.043$$

$$EI\delta = 1.41$$

Final Moments :

$$M_{AB} = -3 + 1.33\,EI\theta_B - 1.33EI\delta = -4.33\,kNm$$

$$M_{BA} = 2.21 \; kNm$$

$$M_{BC} = -2.21 \; kNm$$

$$M_{CB} = 3.32 \; kNm$$

$M_{CD} = -3.32$ kNm

$M_{DC} = -3.25$ kNm

To check equlibrium conditions :

$M_{BA} + M_{BC} = 0$

$2.21 - 2.21 = 0$

$M_{CB} + M_{CD} = 0$

$3.32 - 3.32 = 0$

$2M_{BA} + 2M_{AB} + 3M_{CD} + 3M_{DC} = -24$

$2(-4.33) + 2(2.21) + 3(-3.32) + 3(-3.25) = -24$

$-23.95 = -24$

$-24 = -24$

Hence ok.

Free bending moment :

Span AB = $\frac{wl}{4}$ = 6 kNm

Span BC = $\frac{wl^2}{8}$ = 4.5 kNm

Span CD = 0

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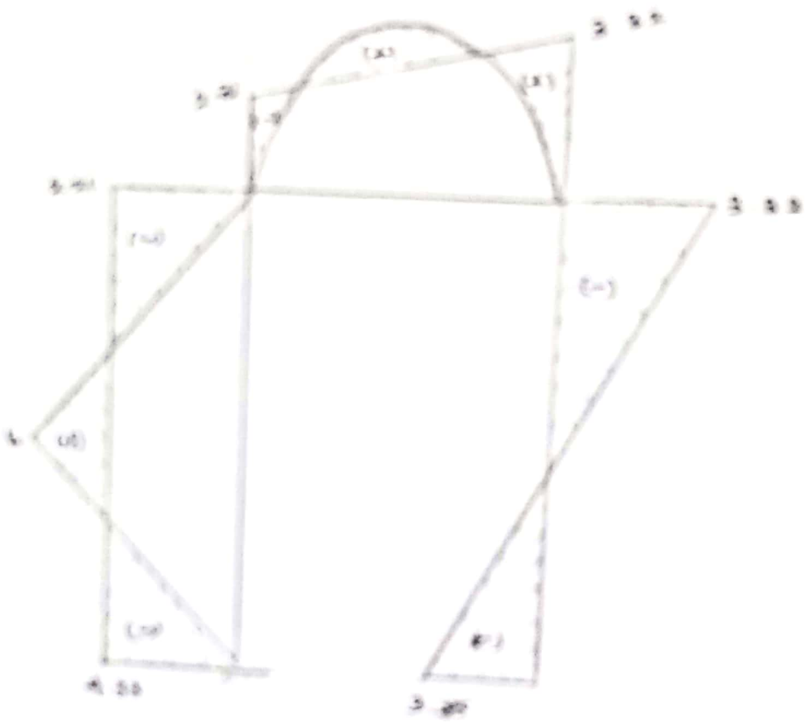


UNIT - III

Moment Distribution Method

* This method is used to analyse the continuous beam and it has been developed by Hardy cross.
* The moment distribution method is applicable for any indeterminate structures like proped cantilever beam, fixed beam, continuous beam Portal frames, sway and non-sway frames etc.,
* This method involves basic concepts such as,

i) Fixed end moments
ii) Relative stiffness
iii) Distribution factor
iv) Carry over factor

Fixed end moments:

The fixed end moments are reaction moments developed in a beam member under certain load conditions with both ends fixed.

Stiffness:

A stiffness for a member can be defined as moment required to produce the unit rotation or slope at a particular point or joint. It is represented by the symbol "K".

* Absolute stiffness

If the stiffness is represented in terms of "E", "L" and "I". Then it is known as absolute stiffness.

* Relative stiffness

If the stiffness is represented in terms of "I" and "L" alone omiting "E". Then it is known as relative stiffness.

Distribution Factor:

The ratio of stiffness of a member to the total stiffness to a joint is known as distribution factor.

$$D_f = \frac{\text{stiffness}}{\text{Total stiffness}}$$

Carry over Factor:

The carry over factor is a ratio of moment developed at ~~fa~~ the end to external or continuous then the carry over factor is 0.5 or 50%..

If the further end is simply supported or roler or hinged the carry over factor is "zero".

Write the stiffness for different end conditions.

For continuous or fixed end $K = \frac{4EI}{l}$

For simply supported or over hanging or hinged,

$$K = 3EI/l$$

How will you check the distribution factor

The Σ of distribution factor at a joint is equal to "1"

Procedure for moment distribution method.

i) Fixed end moments
ii) Stiffness
iii) Distribution Factor
iv) Moment distribution Table
v) Final moments
vi) Equlibrium conditions

Check for equilibrium conditions
Support reactions
BMD and SFD

1. A beam ABC 16m long fixed at the A and C, and continuous over support B carries and UDL of 3kN/m over the span AB and a point load of 10 kN at midspan of BC span AB is 8m and span BC is 8m EI is constant throught out. Analyse the beam using moment distribution method.

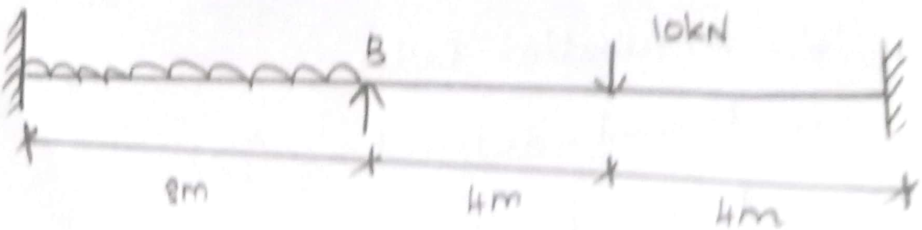
To find:

Fixed End Moments,

$$M_{FAB} = -\frac{wl^2}{12}$$

$$= \frac{(-3\times8^2)}{12}$$

$$= -16\,kNm.$$



$$M_{FBA} = \frac{WL^2}{12}$$

$$= \frac{(3 \times 8^2)}{12} = 16 \text{ kNm}$$

$$M_{FBC} = -\frac{WL}{8}$$

$$= \frac{-10 \times 8}{8} = -10 \text{ kNm}.$$

$$M_{FCB} = \frac{WL}{8}$$

$$= \frac{10 \times 8}{8} = 10 \text{ kNm}.$$

To find stiffness (k):

span AB:

$$k_{AB} = k_{BA} = 4EI/l$$

$$= \frac{4EI}{8}$$

$$= 0.5EI$$

$$k_{BC} = k_{CB} = \frac{4EI}{8}$$

$$= 0.5EI$$

Distribution Factor:

at joint "B":

$$D_{FBA} = \frac{k_{BA}}{k_{BA} + k_{BC}}$$

$$= \frac{0.5EI}{0.5EI + 0.5EI}$$

$$= 0.5EI$$

$DF_{BA} + DF_{BC} = 0.5 + 0.5 = 1.$

Distribution Factor:

Span	AB	B		CB
		BA	BC	
DF		0.5	0.5	
FEM	-16	16	-10	10
Balancing at B		-3	-3	
Carry Over	-1.5			-1.5
Final Moments	-17.5	13	-13	8.5

Final Moments:

$M_{AB} = -17.5,$

$M_{BA} = 13,$

$M_{BC} = -13,$

$M_{CB} = 8.5$

Equlibrium conditions:

At joint "B":

$$\Sigma M_B = 0$$

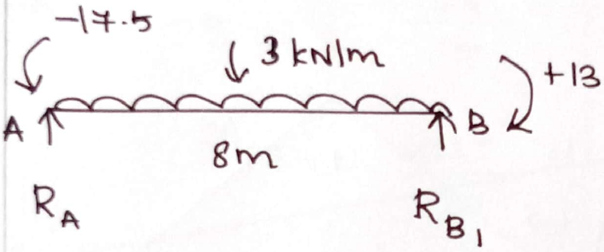
$$M_{BA} + M_{BC} = 0$$

$$13 - 13 = 0$$

Hence checked

Support Reactions :

span AB :



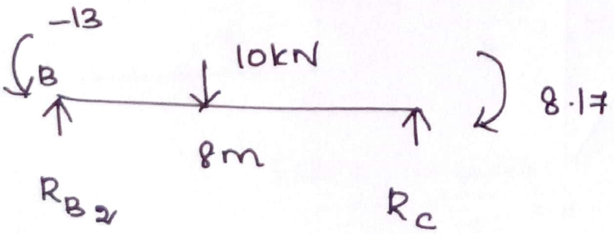
$$(R_A \times 8) - (3 \times 8 \times 8/2) - 17.5 + 13 = 0$$

$$R_A = \frac{-103.5}{8} = 12.56 \text{ kN}$$

$$R_A + R_{B_1} = 3 \times 8 = 11.44 \text{ kN}$$

$$R_{B_1} = 11.44 \text{ kN}$$

span BC :



$$(R_{B_2} \times 8) - (10 \times 4) - 13 + 8.5 = 0$$

$$R_{B_2} = 5.56 \text{ kN}$$

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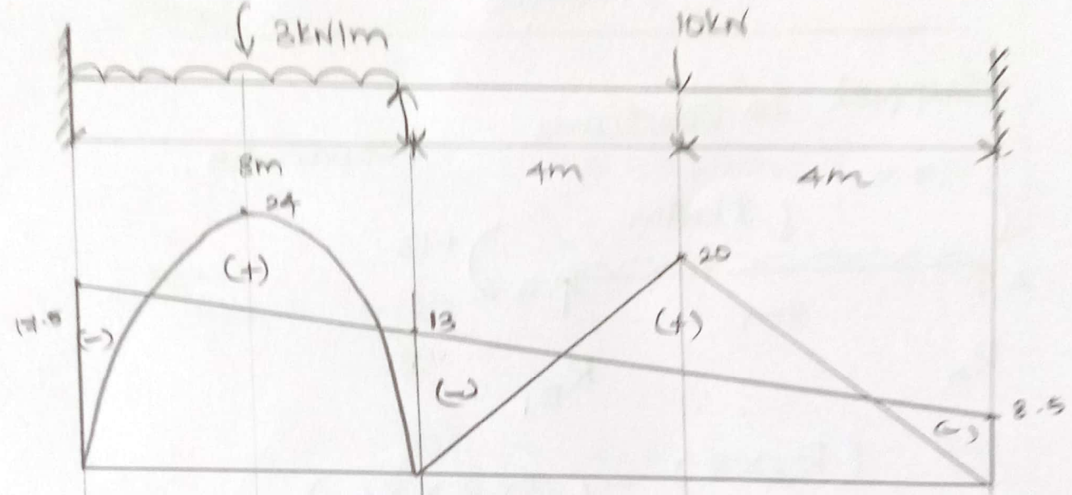
$R_C = 10 - 5.56 = 4.44\ kN$

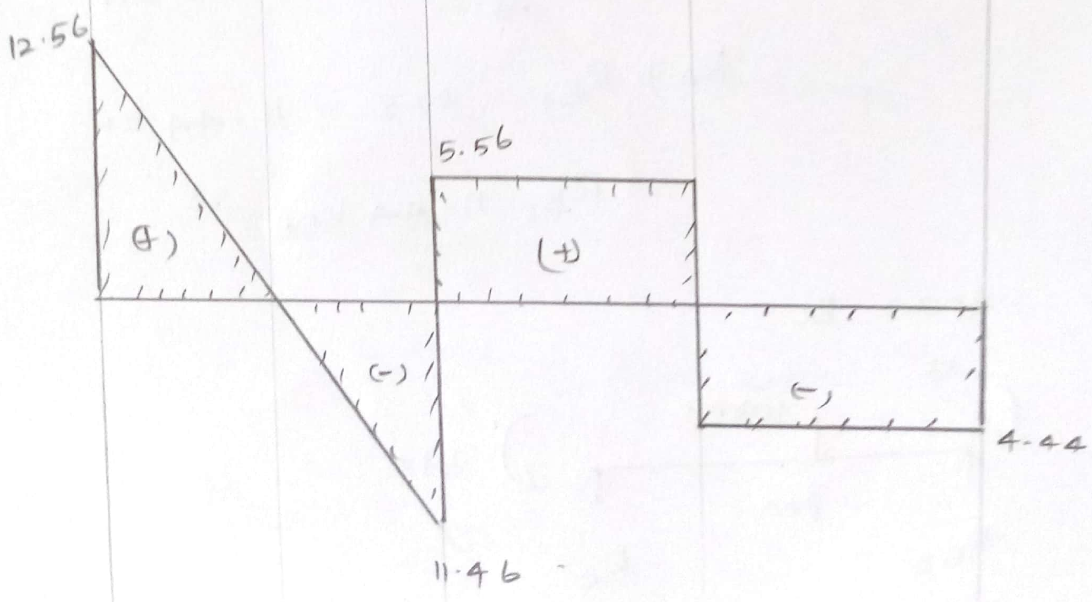
Free BMD:

Span $AB = \frac{wl^2}{8} = \frac{3 \times 8^2}{8} = 24\ kNm$

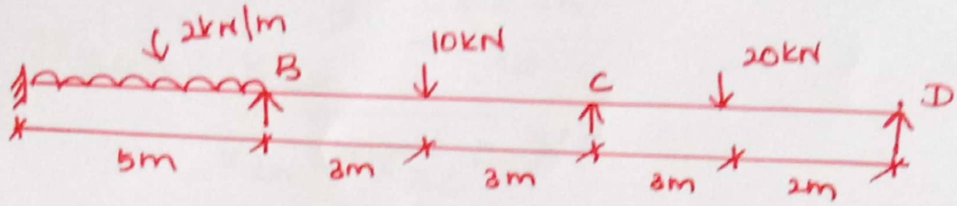
Span $BC = \frac{Wl}{4} = \frac{10 \times 8}{4} = 20\ kNm$



Combined BMD



2) Analyse the continuous beam shown in the figure using moment distribution method. Take EI is constant.



soln:

$$M_{FAB} = -\frac{wl^2}{12}$$

$$= \frac{-2\times5^2}{12} = -4.17 \text{ kNm}$$

$$M_{FBA} = \frac{wl^2}{12}$$

$$= \frac{2\times5^2}{12} = 4.17 \text{ kNm}$$

$$M_{FBC} = -\frac{wl}{8}$$

$$= \frac{-10\times6}{8} = -7.5 \text{ kNm}$$

$$M_{FCB} = \frac{wl}{8}$$

$$= \frac{10\times6}{8} = 7.5 \text{ kNm}$$

$$M_{FCD} = -\frac{wab^2}{l^2}$$

$$= \frac{-20\times3\times2^2}{5^2}$$

$$= -9.6 \text{ kNm}$$

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$$M_{FBC} = \frac{wa^2b}{l^2}$$

$$= \frac{20 \times 3^2 \times 2}{5^2}$$

$$= 14.4 \text{ kNm}$$

To find stiffness "k":

$$k_{AB} = k_{BA} = \frac{4EI}{l}$$

$$= \frac{4EI}{5}$$

$$= 0.8EI$$

Span BC:

$$k_{BC} = k_{CB} = \frac{4EI}{l}$$

$$= \frac{4EI}{6}$$

$$= 0.67EI$$

Span CD:

$$k_{CD} = k_{DC} = \frac{3EI}{l}$$

$$= \frac{3EI}{5}$$

$$= 0.67EI$$

Distribution Factor :

At joint B,

$$DF_{BA} = \frac{K_{BA}}{K_{BA} + K_{BC}}$$

$$= \frac{0.8EI}{0.8EI + 0.67EI}$$

$$= 0.54$$

$$DF_{BC} = \frac{K_{BC}}{K_{BC} + K_{A}}$$

$$= \frac{0.67EI}{0.8EI + 0.8EI}$$

$$= 0.46$$

At joint C,

$$DF_{CB} = \frac{K_{CB}}{K_{CB} + K_{CD}}$$

$$= \frac{0.67EI}{0.67EI + 0.67EI}$$

$$= 0.53$$

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$$DF_{CD} = \frac{K_{CD}}{K_{CB} + K_{CD}}$$

$$= \frac{0.6EI}{0.67\,EI + 0.6EI}$$

$$= 0.47$$

check :

$$DF_{BA} + DF_{BC} = 1 \;;$$

$$DF_{CB} + DF_{CD} = 1$$

Hence Ok.

		B		C		
Span	AB	BA	BC	CB	CD	DC
DF	0	0.54	0.46	0.53	0.47	-
FEM	-4.17	4.17	-7.5	7.5	-9.6	14.4
Release at D. Carry over					-7.2	-14.4
Total moment	-4.17	4.17	-7.5	7.5	-16.8	0
Balancing BC	0.89	(3.33×0.54) 0	-3.33 (3.3×0.46) 1.53	4.93	4.37	0
			2.47	0.77	0	
Balancing BC	-	-1.33	-1.14	-0.41	-0.36	0
Carry over	-0.67	½ 0	-0.21	0.57	0	
Balancing BC	-	0.113	0.96	0.302	0.267	-
Carry over	0.056	0	0.151	0.048	0	0
Final Moments	-3.884	4.753	-4.753	12.57	-12.57	0

Free BMD:

span AB $= \dfrac{wL^2}{8}$

$= \dfrac{2 \times 5^2}{8} = 6.25$ kNm

span BC $= \dfrac{WL}{4}$

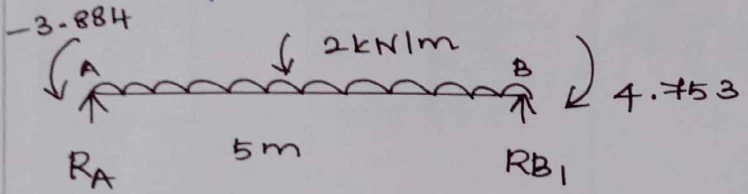
$= \dfrac{10 \times 6}{4} = 15$ kNm

span CD $= \dfrac{Wab}{l}$

$= \dfrac{20 \times 3 \times 2}{5} = 24$ kNm

Reactions:

Taking moment about "B".

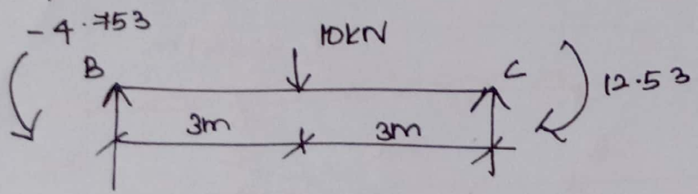


$(R_A \times 5) - (2 \times 5 \times 5/2) - 3.884 + 4.753 = 0$

$R_A = 4.83$ kN , $RB_1 = (2 \times 5) - 4.83$

$RB_1 = 5.17$ kN

span BC:

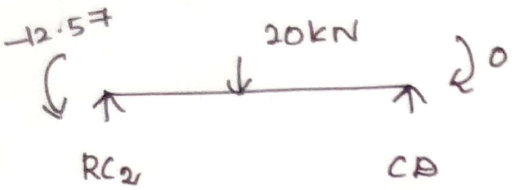


$(RB_2 \times 6) + 10 \times 3 - 4.753 + 12.53 = 0$

$RB_2 = 3.7$ kN

$R_A = 6.3$ kN

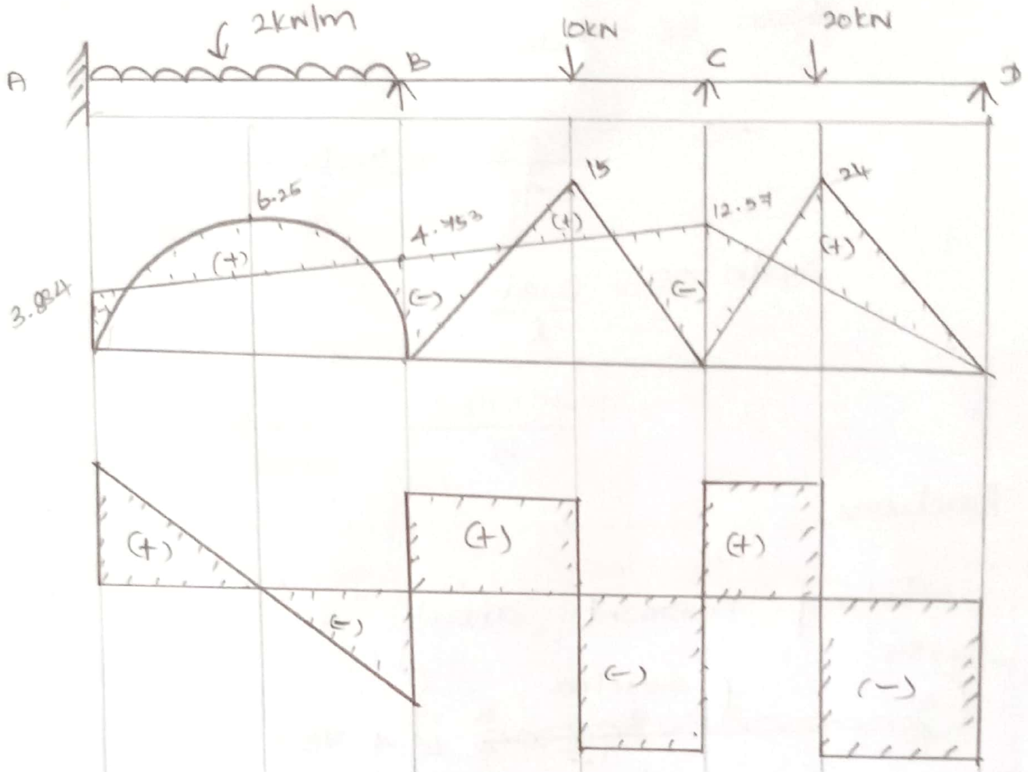
Span CD :



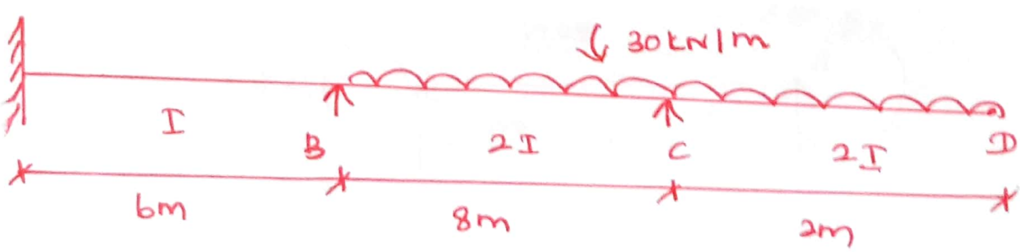
$RC_2 \times 5 - 20 \times 3 - 12.57 = 0$

$RC_2 = 10.51\ kN$

$RD = 9.49\ kN$



3) Draw the SFD and BMD for the given below the sum. By using moment distribution method



soln :

To find fixed end moments.

$M_{FAB} = 0$

$M_{FBA} = 0$

$$M_{FBC} = \frac{-wl^2}{12}$$

$$= -\frac{30 \times 8^2}{12} = -160 \text{ kNm}$$

$$M_{FCB} = \frac{wl^2}{12}$$

$$= \frac{30 \times 8^2}{12} = 160 \text{ kNm}$$

Distribution Factor:

Joints	Member	Stiffness "k"	Total stiffness Σk	Distribution Factor
B	BA	$\frac{4EI}{l} = \frac{4EI}{6}$ $= 0.67EI$	$1.42EI$	$\frac{K}{\Sigma K} = \frac{0.67EI}{0.42EI}$ $= 0.47$
	BC	$\frac{3EI}{l} = \frac{3E(2I)}{8}$ $= 0.75EI$		$\frac{K}{\Sigma K} = \frac{0.75EI}{1.42EI}$ $= 0.53$ $0.47 + 0.53 = 1$ Hence ok.
C	CB	$\frac{4E(2I)}{l}$ $= \frac{4E(2I)}{8}$ $= EI$	EI	$\frac{K}{\Sigma K} = \frac{EI}{EI} = 1$ $K/\Sigma K = 0/\Sigma I = 0$ $1 + 0 = 1$ Hence ok
	CD	0	–	

Span	AB	B		C		DC
		BA	BC	CB	CD	
DF	-	0.47	0.53	1	0	-
FEM	0	0	-160	160	-60	-
Balancing at C				-100		
Carry over			-50			
Initial Moments	0	0	-210	60	-60	-
Balancing at B		98.7	111.3			
Carry over	49.35					
Final Moments	49.35	98.7	-98.7	60	-60	-

Free BMD:

Span AB = 0

Span BC $= \frac{wl^2}{8}$

$= \frac{30 \times 8^2}{8} = 240$ kNm

Span CD $= -\frac{wl^2}{8}$

$= \frac{-30 \times 2^2}{2}$

$= -60$ kNm

$M_{AB} = 49.35$ kNm

$M_{BA} = 98.7$ kNm

$M_{BC} = -98.7$ kNm

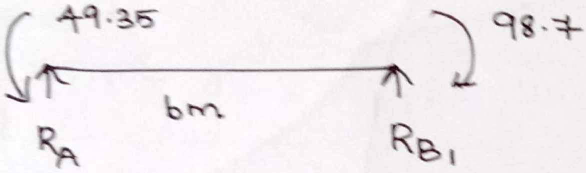
$M_{CB} = 60$ kNm

$M_{CD} = -60$ kNm

$M_{DC} = -$

Reactions:

For span AB:



Moment about R_{B1},

$$6R_A + 49.35 + 98.7 = 0$$

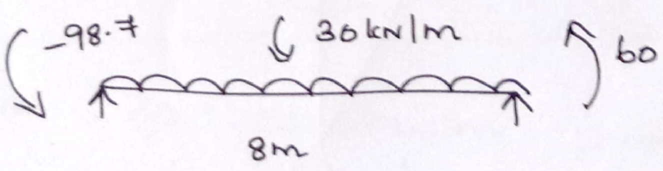
$$6R_A = -148.05$$

$$R_A = -24.67 \text{ kN}$$

$$R_A + R_{B_1} = 0$$

$$R_{B_1} = 24.67 \text{ kN}$$

For span BC:

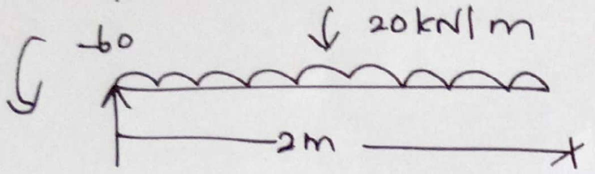


$$R_{B_2} \times 8 + (30 \times 8 \times 8/2) - 98.7 + 60 = 0$$

$$R_{B_2} = 124.84 \text{ kN}$$

$$R_A = 115.2 \text{ kN}$$

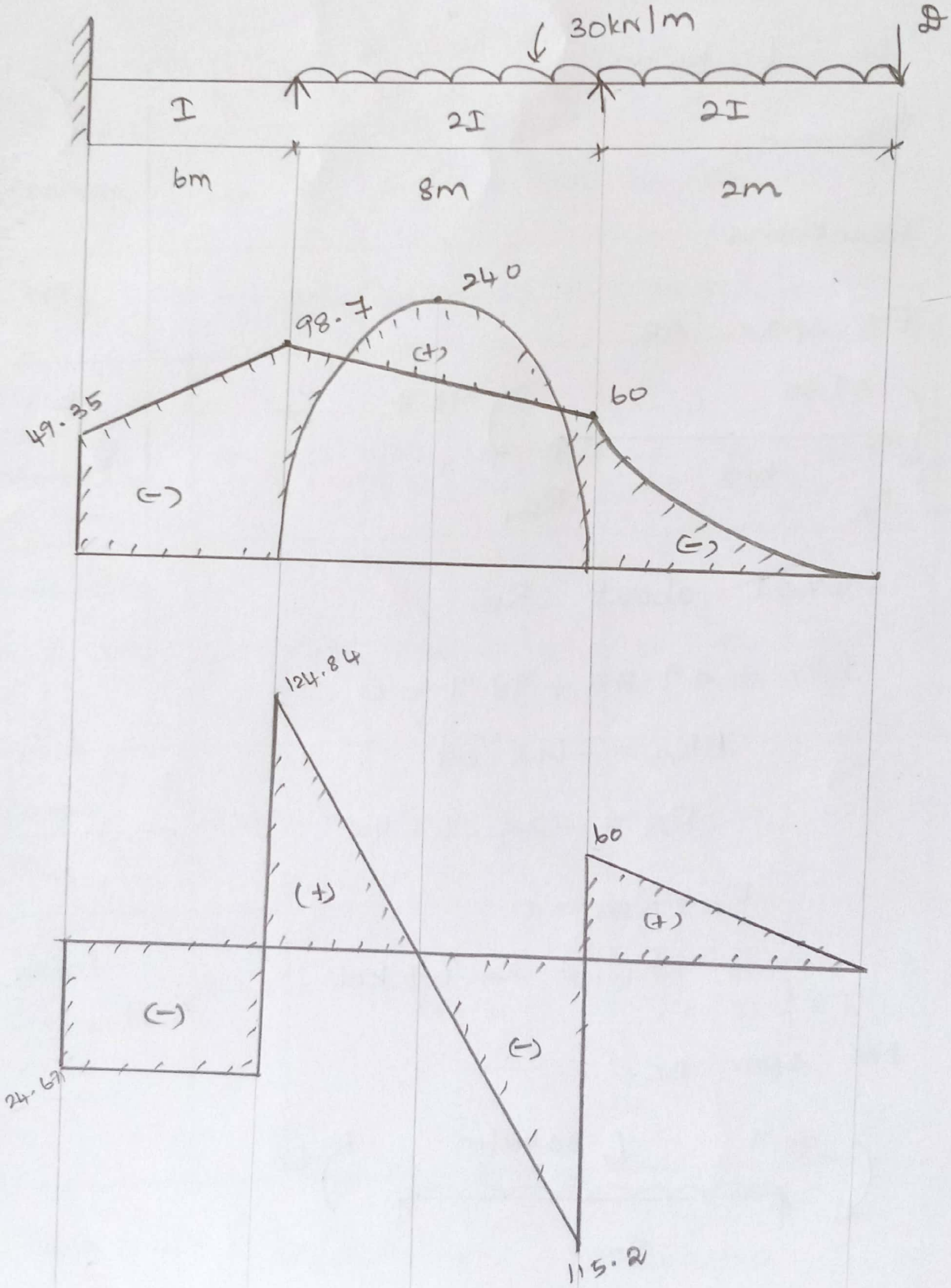
For span CD:



$R_{C_2} = w \times l$

$= 30 \times 2$

$= 60\ kN$



$6.87 - 6.87 = 0$

Hence OK.

At joint "B" :-

$M_{BA} + M_{BC} + M_{BE} = 0$

$27.81 - 27.81 = 0$

Hence OK.

At joint "C" :

$M_{CB} + M_{CF} = 0$

$6.87 - 6.87 = 0$

Hence OK.

To find Free BMD:

span $AB = wL^2/8$

$= \frac{10 \times 5^2}{8} = 31.25$ kNm

span $BC = \frac{wL^2}{8}$

$= 10 \times 5^2/8 = 31.25$ kNm

span $AD = 0$

span $BE = 0$

span $CF = 0$

C	CB	$\frac{4EI}{l} = \frac{4E(2I)}{5}$ $= 1.6EI$ $\frac{4EI}{l} = \frac{4EI}{5}$ $= 0.8EI$	$1.6EI$ $+$ $0.8EI$ $= 2.4 EI$	$\frac{1.6EI}{2.4EI} = 0.67$ $\frac{0.8EI}{2.4EI} = 0.33$ $0.67 + 0.33 = 1$ Hence OK.

Joint	D	A		B			C		F	E
Member	DA	AD	AB	BA	BE	BC	CB	CF	FC	EB
DF	–	0.33	0.67	0.33	0.33	0.33	0.67	0.33	–	–
Fixed End Moments	0	0	-20.83	20.83	0	-20.83	20.83	0	0	0
Balancing A, B and C	–	6.87	13.96	0	0	0	-13.96	-6.87	–	–
Carry Over	3.43	–	0	6.98	–	-6.98	0	–	-3.43	–
Final Moments	3.43	6.87	-6.87	27.81	0	-27.87	6.87	-6.87	-3.43	0

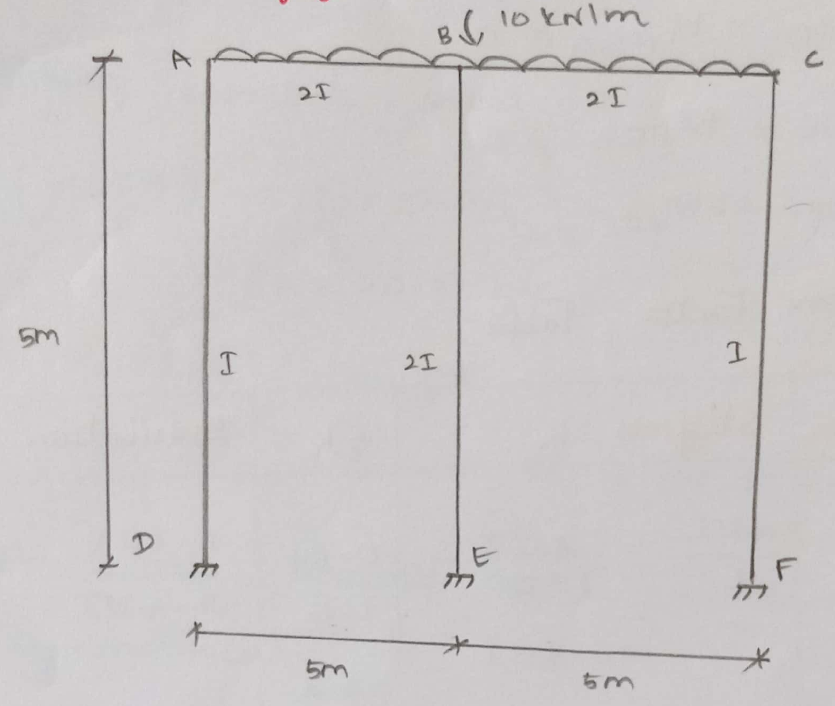
Check for equilibrium conditions:

At joint "A":

$$M_{AD} + M_{AB} = 0$$

Analysis of portal frame by moment distribution method

Analyse the frame by moment distribution method shown in figure.



To find fixed end moments:

$$M_{FAB} = \frac{-wL^2}{12}$$

$$= \frac{-10 \times 5^2}{12} = -20.83 \text{ kNm}$$

$$M_{FBA} = \frac{wL^2}{12}$$

$$= \frac{10 \times 5^2}{12} = 20.83 \text{ kNm}$$

$$M_{FBC} = \frac{wl^2}{12}$$

$$= \frac{10 \times 5^2}{12} = 20.83 \text{ kNm}$$

$$M_{FBC} = -\frac{wl^2}{12}$$

$$= -10 \times 5^2 / 12$$

$$= -20.83 \text{ kNm}$$

$$M_{FAB} = M_{FBA} = 0$$

$$M_{FBE} = M_{FEB} = 0$$

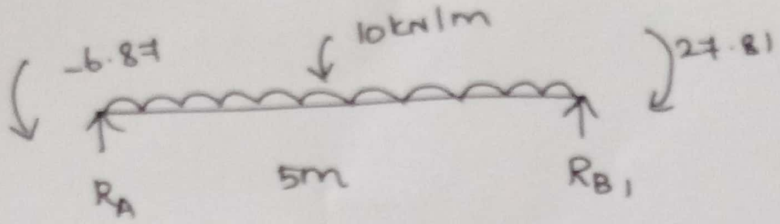
$$M_{FCF} = M_{FFC} = 0$$

Distribution Factor Table:

Joint	Member	Stiffness K	Σk	Distribution factor
A.	AD	$\frac{4EI}{l} = \frac{4EI}{5}$ $= 0.8EI$	$0.8 + 1.6$ $= 2.4 EI$	$\frac{0.8EI}{2.4EI} = 0.33$
	AB	$\frac{4EI}{l} = \frac{4EI}{5}$ $= 1.6EI$		$\frac{0.8EI}{2.4EI} = 0.66$ $0.66 + 0.33 = 1$ Hence OK.
B.	BA	$\frac{4EI}{l} = \frac{4E(2I)}{5}$ $= 1.6 EI$	$1.6EI + 1.6EI + 1.6EI$ $= 4.8 EI$	$\frac{1.6EI}{4.8EI} = 0.33$
	BE	$\frac{4EI}{l} = \frac{4E(2I)}{5}$ $= 1.6EI$		$\frac{1.6EI}{4.8EI} = 0.3$
	BC	$\frac{4EI}{l} = \frac{4E(2I)}{5}$ $= 1.6EI$		$\frac{1.6EI}{4.8EI} = 0.33$ $0.33 + 0.33 + 0.33$ $\simeq 1$. OK.

Find Reactions:

Span AB:



Taking moment about "B",

$$(R_A \times 5) - (10 \times 5 \times 5/2) - 6.944 + 27.81 = 0$$

$$R_A = 20.81 \text{ kN}$$

$$(R_A + R_{B1}) = 10 \times 5$$

$$R_{B1} = 50 - 20.81 = 29.19 \text{ kN}$$

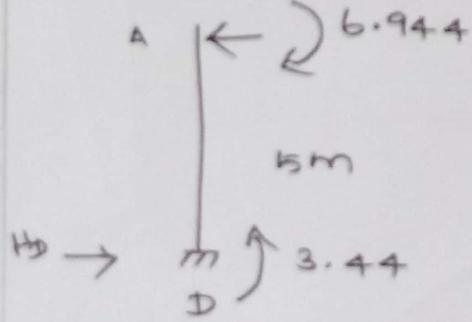
As the frame and loading are symmetrical.

For span BC;

Hence, $RB_2 = 29.19$ kN

$RC_1 = 20.81$ kN

Span AD:



$$-(H_A \times 5) + 6.88 + 3.44 = 0$$

$$H_A = 2.06 \text{ kN}$$

$$H_D = -2.06 \text{ kN}$$

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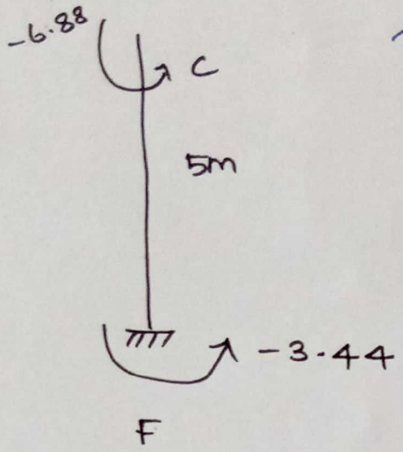
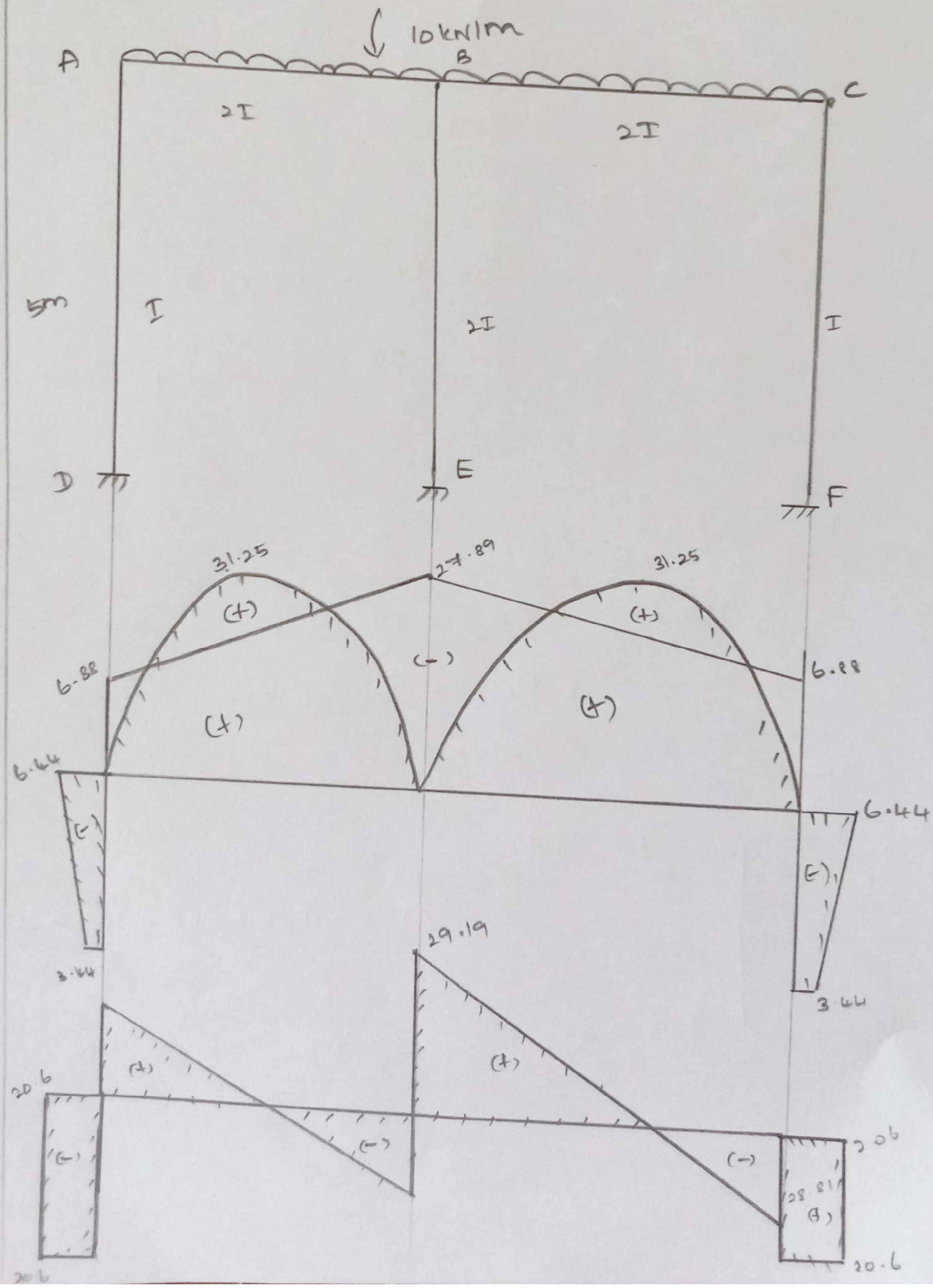
span CF :



similarly to span AD,

$H_C = 2.06$ kN

$H_F = -2.06$ kN



Unit - 4

Flexibility Matrix Method

⇒ With the advent of computers; matrix method of solving structures have become very popular.

⇒ In an elastic structures are two sets of interrelated quantities.

⇒ Forces (including moments, stresses, reaction etc.,)

⇒ Displacements (including rotations, strains, twists etc.,)

These two are termed as generalised forces and generalised co-ordinates.

Two kinds of Matrix Methods :

⇒ The flexibility method (or) force method (or) compactability method.

⇒ The stiffness method (or) Displacement method (or) Equlibrium method.

Flexibility Method :

⇒ Each element of a flexibility matrix [(a) Matrix (or) (α) matrix], represents a a displacement at a co-ordinate (i) due to unit force at a co-ordinate (j)

⇒ That element of (α) can be called Flexibility co-efficient (α_{ij})

⇒ If the (α) matrix of a structure is known. we can say the behaviour of the structure.

Degree of static Indeterminancy:

The number of equations required over and above the equations of the static equlibrium for the analysis of structure is known as degree of static indeterminancy of the structure.

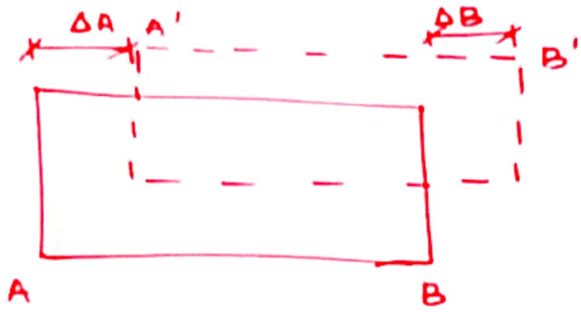
Degree of kinematic Indeterminancy:

The number of Equlibrium conditions needed to find the displacement components of all joints of the structure are known as the Degree of kinematic Indeterminancy (or) Degree of freedom of the structure.

Degree of Freedom :

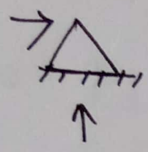
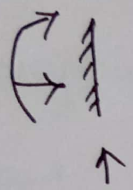
→ It can be defined as the independent joint deformation.

→ The freedom for the moment in the x, y and rotational directions of every significant point in a structure is an important parameter in a structure.



In this figure A and B move axially that is two degree of freedom but actually is A B cannot deform (rigid body). We only have one degree of freedom that means

$$\Delta_A = \Delta_B$$

Plane	Restraint	Releases
	1	2
	2	1
	3	0

Equlibrium Conditions :

Three conditions of Equlibrium that is sum of vertical forces, Horizontal forces and moment at any point or joint is zero

(i) $\Sigma H = 0$

(ii) $\Sigma V = 0$

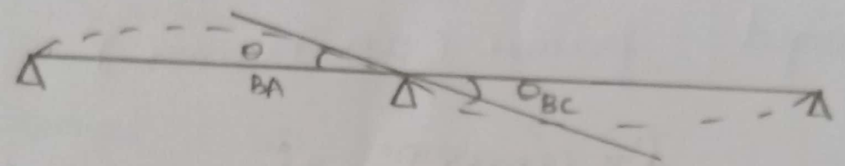
(iii) $\Sigma M = 0$

Compatability :

Compatability in similar in concept to equlibrium frees should be in equlibrium.

Displacement of structure should be compatible

Compatability conditions:



Let us suppose it deflects as shown in BA will have a slope θ_{BA}. BC will have a slope θ_{BC}. But the beam ABC is continuous and there hinged at B. so ABC can have only are slope at B. Hence, $\theta_{BA} = \theta_{BC}$ This is a compatability condition.

i) Determinate structures

ii) Indeterminate structures, Additional conditions are required.

Formula for degree of Indeterminancy:-

Two dimensional pin joint truss (2D Truss).

$$I = (m+r) - 2j$$

where,

m = No of members

r = No of reactions

I = Indeterminancy

j = No of joints

⇒ Two dimensional rigid frames/ Plane rigid frames (2D Frames)

$$I = (3m+r) - 3j$$

⇒ Three dimensional space truss (3D truss).

$$I = (m+r) - 3j$$

⇒ space frames (3D Frames)

External and Internal Indeterminancies :-

⇒ Internal indeterminancy is the excess number of internal forces present in an member that make structure indeterminate.

⇒ External Indeterminancy is the excess number of external reactions in a member that make a structure indeterminate.

$$\text{Indeterminancy } (i) = I \cdot I + E \cdot I$$

$$E \cdot I = r - e$$

where,

r = Number of support reactions

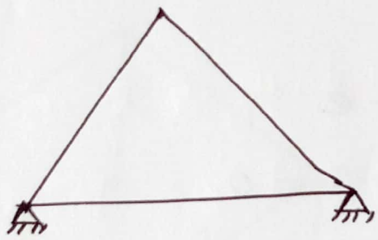
e = Equlibrium conditions

$$I.I = i - E.I$$

Space Frames (3D frames)

$$I = (6m + r) - 6j$$

1) Find the indeterminancy (i) Internal Indeterminancy (I.I), External Indeterminancy (E.I) for the given structures.



$$I = (m + r) - 2j$$

No of members $m = 3$

No of reactions $r = 2 \times 2 = 4$

No of joint $j = 3$

$$I = (3 + 4) - 2(3)$$

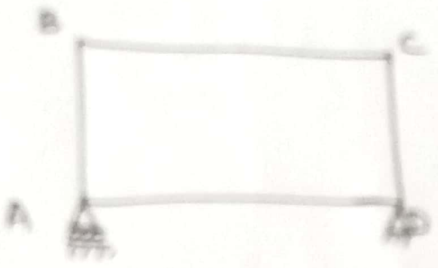
$$= 1$$

External Indeterminancy $= r - e$

$= 4 - 3 = 1$

Internal Indeterminacy $= 1 - 1 = 0$

2)



$I = (3m + r) - 3j$

$m = 4,$

$r = 3 + 0 = 3$

$j = 4$

$I = 3(4) + 3 - 3(4)$

$= 3$

$EI = r - e$

$= 3 - 3 = 0$

$I = EI + II$

$I = 0 + 3 = 3$

3)

$I = (m+r) - 3j$

$m = 3$

$r = 6 \times 3 = 18$

$j = 4$

$I = (3+18) - 3(4)$

$I = 12$

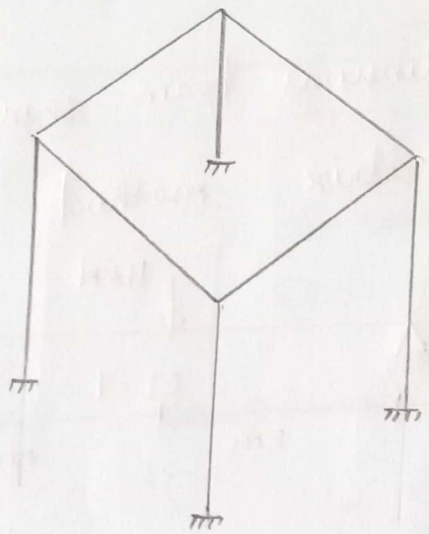
$I = E.I + I.I$

$I.I = I - E.I$

$= 9 - 12$

$I.I = -3$

4)



$I = (6m+r) - 6j$

$m = 8$

$r = 4 \times 6 = 24$

$j = 8$

$I = [(6\times8)+24] - (6\times8)$

$i = 24$

$E.I = r - e$

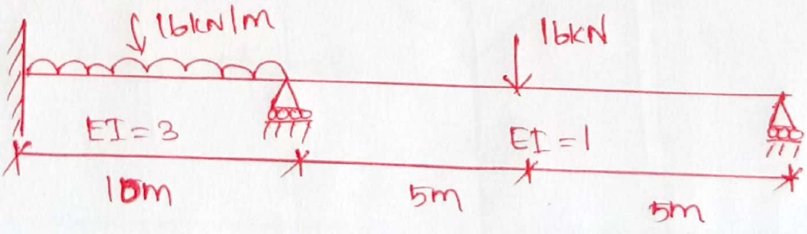
$= 24 - 6 = 18$

$I.I = 24 - 18 = 6$

Primary structure :

A structure found by removing the excess or redundant restraints from an indeterminate structure making it an statically determinate is called Primary structure.

1) Analyse the continuous beam shown in figure by Flexibility matrix method



step 1 : To find statically indeterminancy :-

$SI = r - e$

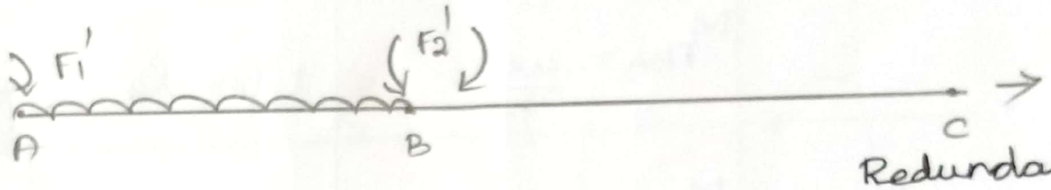
where, r = No of supports reactions = 4

e = Equlibrium conditions = 2

[∵ For continuous beam hzt reaction is neglected]

Step 2 : Formation of "b" matrix

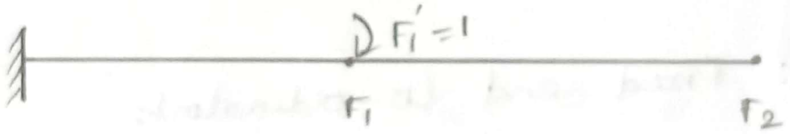
If consider "a" and "b" as redundants to form primary structure,



Redundant system

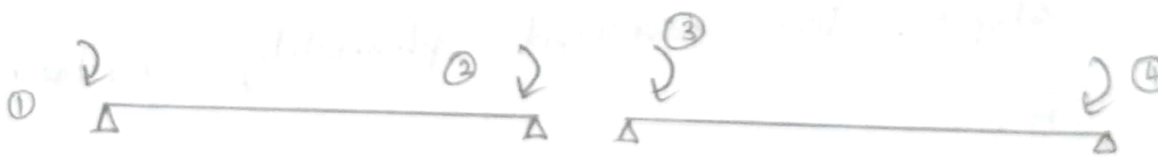
Introduce hinges at A & B

System co-ordinate :



To consider SS (or) roller support (or) hinged support only.

Element co-ordinate :



$$b' = \begin{bmatrix} 1 & 0 \\ 0 & -1 \\ 0 & 1 \\ 0 & 0 \end{bmatrix}$$

$$b^o = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 1 & 0 \\ 0 & 1 \end{bmatrix}$$

Span	Name of Moment	FEM (P^o)	Equivalent Joint Load
AB	$M_{FAB} = \frac{-WL^2}{12}$	-133.33	133.33
	$M_{FBA} = \frac{WL^2}{12}$	$+133.33$	-133.33
BC	$M_{FBC} = -\frac{WL}{8}$	-20	$+20$
	$M_{FCB} = \frac{WL}{8}$	$+20$	-20

Step 4: Fixed end co-ordinators:

$$F^o = \begin{bmatrix} -133.33 + 20 \\ 20 \end{bmatrix} = \begin{bmatrix} -113.33 \\ 20 \end{bmatrix}$$

Step 5: The assembled flexibility matrix (α) is,

$$\alpha = \frac{l}{6EI}\begin{pmatrix} 2 & -1 \\ -1 & 2 \end{pmatrix} \text{ for 1 element}$$

for 2 element 4×4

for 3 element 6×6

$$\alpha = \frac{l}{6EI}\begin{pmatrix} 2 & -1 & 0 & 0 \\ -1 & 2 & 0 & 0 \\ 0 & 0 & 2 & -1 \\ 0 & 0 & -1 & 2 \end{pmatrix}$$

$$= \frac{1}{6EI}\begin{pmatrix} 2l & -l & 0 & 0 \\ -l & 2l & 0 & 0 \\ 0 & 0 & 2l & -l \\ 0 & 0 & -l & 2l \end{pmatrix}$$

$$= \frac{1}{6}\begin{bmatrix} 2l/3 & -l/3 & 0 & 0 \\ -l/3 & 2l/3 & 0 & 0 \\ 0 & 0 & 2l & -l \\ 0 & 0 & -l & 2l \end{bmatrix}$$

$$\alpha = \begin{bmatrix} l/9 & -l/18 & 0 & 0 \\ -l/18 & l/9 & 0 & 0 \\ 0 & 0 & l/3 & -l/6 \\ 0 & 0 & -l/6 & l/3 \end{bmatrix}$$

step 6: To find F' (Redundant Forces)

$$F' = -(a)^{-1}(a_{10})(F^{0})$$

$$a_{11} = (b')^{T}(\alpha)(b')$$

$$a_{11} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 1 & 0 \end{bmatrix} \begin{bmatrix} l/9 & -l/18 & 0 & 0 \\ -l/18 & l/9 & 0 & 0 \\ 0 & 0 & l/3 & -l/6 \\ 0 & 0 & -l/6 & l/3 \end{bmatrix} \times \begin{bmatrix} 1 & 0 \\ 0 & -1 \\ 0 & 1 \\ 0 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} l/9+0+0+0 & -l/18+0+0+0 & 0 & 0 \\ l/18 & -l/9 & l/3 & -l/6 \end{bmatrix} \times \begin{bmatrix} 1 & 0 \\ 0 & -1 \\ 0 & 1 \\ 0 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} l/9+0+0+0 & 0+l/18+0+0 \\ l/18+0+0+0 & 0+l/9+l/3+0 \end{bmatrix}$$

$$a_{11} = \begin{bmatrix} l/9 & l/18 \\ l/18 & \frac{3l+9l}{27} \end{bmatrix} = \begin{bmatrix} l/9 & l/18 \\ l/18 & \frac{12l}{27} \end{bmatrix} = \begin{bmatrix} 1.1 & 0.555 \\ 0.555 & 4.444 \end{bmatrix}$$

$$(a_{11})^{-1} = \begin{pmatrix} 0.96 & -0.12 \\ -0.12 & 0.24 \end{pmatrix}$$

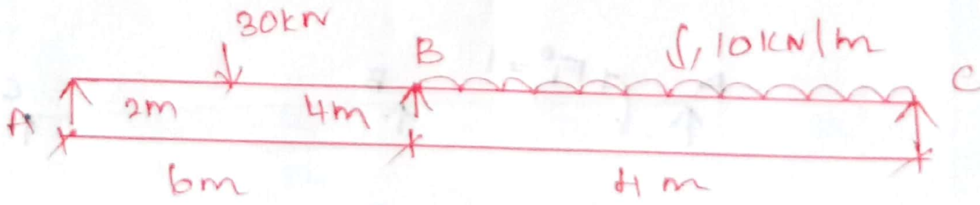
$$(a_{10}) = (b')^T (\alpha)(b$$

$$= \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 1 & 0 \end{bmatrix} \begin{bmatrix} l/9 & -l/18 & 0 & 0 \\ -l/18 & l/9 & l/3 & 0 \\ 0 & 0 & l/3 & -l/6 \\ 0 & 0 & -l/6 & l/3 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & -1 \\ 0 & 1 \\ 0 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} \frac{l}{9} & -l/18 & 0 & 0 \\ \frac{l}{18} & -l/9 & l/3 & -l/6 \end{bmatrix} \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$a_{10} = \begin{bmatrix} 0 & 0 \\ l/3 & -l/6 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 3.33 & -1.66 \end{bmatrix}$$

2) Analyse the continuous beam shown in the figure by flexibility matrix method.



soln:

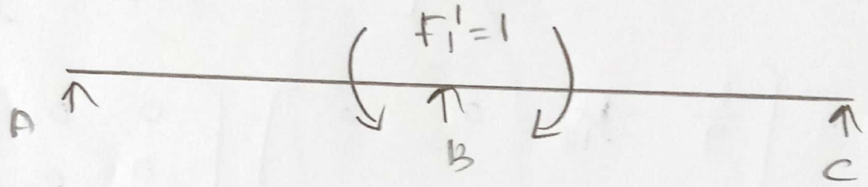
step 1 : static Indeterminancy :

$$SI = r - e$$
$$= 3 - 2$$
$$= 1$$

The degree of redundancy = 1

Consider support B as redundant.

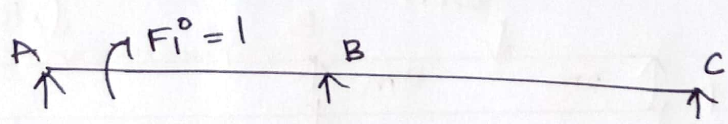
step 2 : Redundant system :



$$b' = \begin{bmatrix} 0 \\ -1 \\ 1 \\ 0 \end{bmatrix}$$

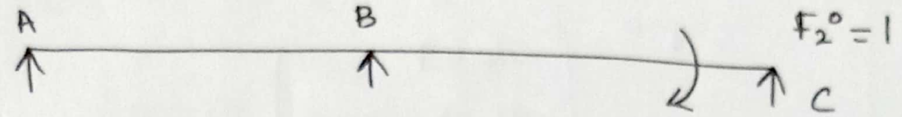
system co-ordinate :

Applying $F_1^{\circ} = 1$



$$b^{\circ} = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

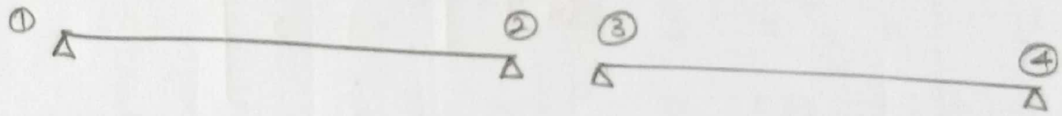
Applying $F_2^\circ = 1$



$$b^\circ = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

$$\therefore \quad b^\circ = \begin{pmatrix} 1 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 1 \end{pmatrix}$$

Element coordinates



Span	Name of Moments	FEM	Equivalent Joint Loads
AB	$M_{FAB} = -\frac{Wab^2}{l^2}$	-26.67	26.67
	$M_{FBA} = \frac{Wa^2b}{l^2}$	13.33	-13.33
BC	$M_{FBC} = -\frac{wl^2}{12}$	-13.33	13.33
	$M_{FCB} = \frac{wl^2}{12}$	13.33	-13.33

step 4: To find F^o (only simply supported)

$$F^o = \begin{bmatrix} 26.67 \\ -13.33 + 13.33 \\ -13.33 \end{bmatrix}$$

$$F^o = \begin{bmatrix} 26.67 \\ 0 \\ -13.33 \end{bmatrix}$$

step 5: To find α

Assemble Flexibility Matrix,

$$\alpha = \frac{l}{6EI}\begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix}$$

$$= \frac{l}{6EI}\begin{bmatrix} 2 & -1 & 0 & 0 \\ -1 & 2 & 0 & 0 \\ 0 & 0 & 2 & -1 \\ 0 & 0 & -1 & 2 \end{bmatrix}$$

$$= \frac{1}{6EI}\begin{pmatrix} 2l & -l & 0 & 0 \\ -l & 2l & 0 & 0 \\ 0 & 0 & 2l & -l \\ 0 & 0 & -l & 2l \end{pmatrix}$$

$$= \frac{1}{6EI}\begin{bmatrix} 12 & -6 & 0 & 0 \\ -6 & 12 & 0 & 0 \\ 0 & 0 & \overset{8}{\cancel{12}} & -4 \\ 0 & 0 & -4 & \underset{8}{\cancel{12}} \end{bmatrix}$$

$$= \frac{1}{EI}\begin{bmatrix} 2 & -1 & 0 & 0 \\ -1 & 2 & 0 & 0 \\ 0 & 0 & 1.33 & -0.67 \\ 0 & 0 & -0.67 & 1.33 \end{bmatrix}$$

step 6 : To find F' :

$$F' = -(a_{11})(a_{10})(F^0)$$

$$a_{11} = (b')^T(\alpha)(b')$$

$$a_{10} = b'(T)(\alpha)(b^0)$$

$$a_{11} = (b')^T(\alpha)(b')$$

$$= \begin{bmatrix} 0 & -1 & 1 & 0 \end{bmatrix} \frac{1}{EI}\begin{bmatrix} 2 & -1 & 0 & 0 \\ -1 & 2 & 0 & 0 \\ 0 & 0 & 1.33 & -0.67 \\ 0 & 0 & -0.67 & 1.33 \end{bmatrix}\begin{bmatrix} 0 \\ -1 \\ 1 \\ 0 \end{bmatrix}$$

$$= \frac{1}{EI}\begin{bmatrix} 1 & -2 & 1.33 & -0.67 \end{bmatrix}\begin{bmatrix} 0 \\ -1 \\ 1 \\ 0 \end{bmatrix}$$

$$a_{11} = \frac{1}{EI}(2 + 1.333)$$

$$= 3.33/EI$$

$$(a_{11})^{-1} = 0.300\,EI$$

$$a_{10} = (b')^T (\alpha)(b^o)$$

$$= \frac{1}{EI}\begin{pmatrix} 1 & -2 & 1.33 & -0.67 \end{pmatrix}\begin{bmatrix} 1 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 1 \end{bmatrix}$$

$$a_{10} = \frac{1}{EI}(1 - 0.67)$$

$$F' = -0.300EI \times \frac{1}{EI}(1 - 0.67)\begin{pmatrix} 26.67 \\ -13.33 \end{pmatrix}$$

$$= (-0.300 + 0.201)\begin{pmatrix} 26.67 \\ -13.33 \end{pmatrix}$$

$$F' = -10.68$$

Step 7 : End Moments :

To find "p"

$$P = (b^o)(F^o) + (b')(F') + p^o$$

$$= \begin{bmatrix} 1 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 26.67 \\ -13.33 \end{bmatrix} + \begin{bmatrix} 0 \\ -1 \\ 1 \\ 0 \end{bmatrix}(-10.68) + \begin{bmatrix} -26.68 \\ 13.33 \\ -13.33 \\ 13.33 \end{bmatrix}$$

$$= \begin{bmatrix} 26.67 \\ 0 \\ 0 \\ -13.33 \end{bmatrix} + \begin{bmatrix} 0 \\ 10.68 \\ -10.68 \\ 0 \end{bmatrix} + \begin{bmatrix} -26.67 \\ 13.33 \\ -13.33 \\ 13.33 \end{bmatrix}$$

$$P = \begin{bmatrix} 0 \\ 24.01 \\ -24.01 \\ 0 \end{bmatrix}$$

$M_{AB} = 0$

$M_{BA} = 24.01$ kNm

$M_{BC} = -24.01$ kNm

$M_{CB} = 0$

step 8 : Free BMD

Span AB $= \dfrac{wab}{l}$

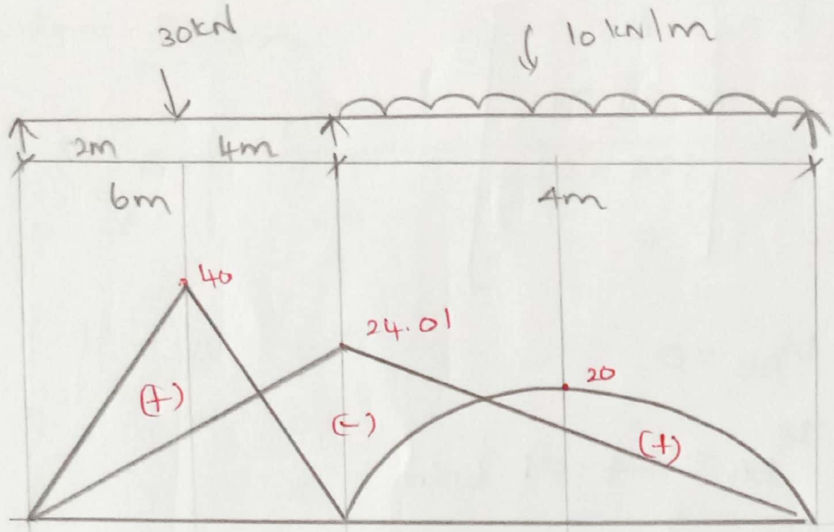
$= \dfrac{30 \times 2 \times 4}{6}$

$= 40$ kNm

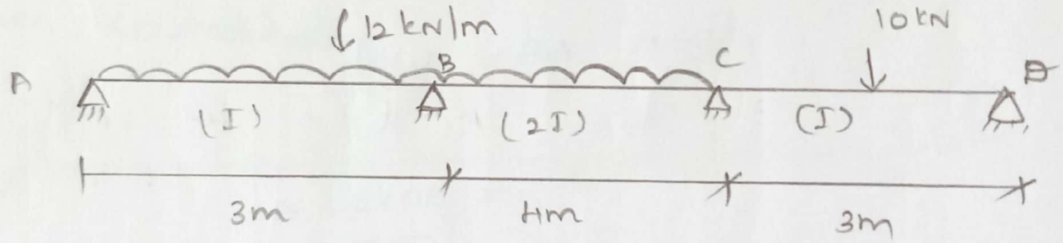
Span BC $= \dfrac{wl^2}{8}$

$= \dfrac{10 \times 4^2}{8}$

$= 20$ kNm



3) Analyse the continuous beam shown in figure using Flexibility method.



soln:

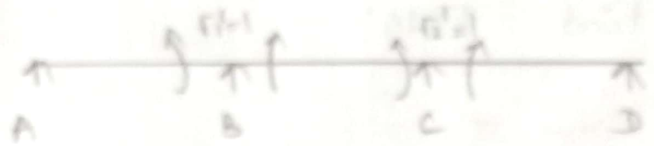
Step 1: To find statically indeterminacy:

$$SI = r - e$$
$$= 4 - 2$$
$$= 2$$

The degree of redundancy is 2

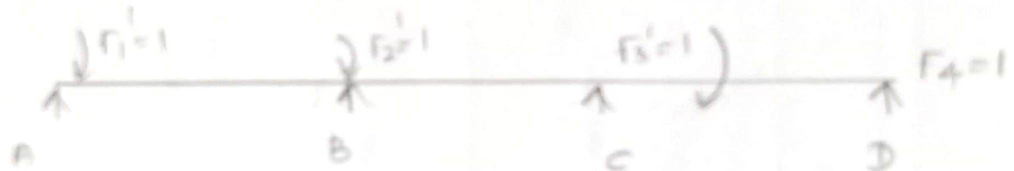
Step 2: consider B and C as redundants

Redundant system:



$$b' = \begin{bmatrix} 0 & 0 \\ -1 & 0 \\ 1 & 0 \\ 0 & -1 \\ 0 & 1 \\ 0 & 0 \end{bmatrix}$$

System co-ordinates:



$$b^0 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Element co-ordinates:

To Find FEM:

Span	Name of Moments	FEM	Equivalent Joint Loads
AB	$M_{FAB} = -\frac{WL^2}{12}$	-9	9
	$M_{FBA} = \frac{Wl^2}{12}$	$+9$	-9
BC	$M_{FBC} = -\frac{WL^2}{12}$	$+16$	$+16$
	$M_{FCB} = \frac{WL^2}{12}$	$+16$	-16
CD	$M_{FCD} = -\frac{Wl}{8}$	-6	$+6$
	$M_{FDC} = \frac{Wl}{8}$	$+6$	-6

step 4:

To find F^o (only simply supported)

$$F^o = \begin{bmatrix} 9 \\ 7 \\ -10 \\ -6 \end{bmatrix}$$

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step 5: To find α:

Assembled Flexibility Matrix:

$$\alpha = \frac{l}{6EI}\begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix}$$

$$= \frac{l}{6EI}\begin{bmatrix} 2 & -1 & 0 & 0 & 0 & 0 \\ -1 & 2 & 0 & 0 & 0 & 0 \\ 0 & 0 & +2 & -1 & 0 & 0 \\ 0 & 0 & -1 & 2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 2 & -1 \\ 0 & 0 & 0 & 0 & -1 & 2 \end{bmatrix}$$

$$= \frac{1}{EI}\begin{bmatrix} \frac{2l}{6EI} & -l/6 & 0 & 0 & 0 & 0 \\ \frac{-l}{6} & 2/6\,l & 0 & 0 & 0 & 0 \\ 0 & 0 & 2l/6 & -l/6 & 0 & 0 \\ 0 & 0 & -l/6 & \frac{2l}{6} & 0 & 0 \\ 0 & 0 & 0 & 0 & 2l/6 & -l/6 \\ 0 & 0 & 0 & 0 & -l/6 & 2l/6 \end{bmatrix}$$

$$\alpha = \frac{1}{EI}\begin{bmatrix} 1 & -0.5 & 0 & 0 & 0 & 0 \\ -0.5 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0.67 & -0.33 & 0 & 0 \\ 0 & 0 & -0.33 & 0.67 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & -0.5 \\ 0 & 0 & 0 & 0 & -0.5 & 1 \end{bmatrix}$$

Scanned with OKEN Scanner

step 6: To Find f':

$$F' = (a_{11})(a_{10})(F_0^{\circ})$$

$$a_{11} = (b')^T (\alpha)(b')$$

$$a_{10} = (b')^T (\alpha)(b^{\circ})$$

$$a_{11} = \begin{bmatrix} 0 & -1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 1 & 0 \end{bmatrix} \frac{1}{EI} \begin{bmatrix} 1 & -0.5 & 0 & 0 & 0 & 0 \\ -0.5 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0.67 & -0.33 & 0 & 0 \\ 0 & 0 & -0.33 & 0.67 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & -0.5 \\ 0 & 0 & 0 & 0 & -0.5 & 1 \end{bmatrix} \frac{1}{I}$$

$$\begin{bmatrix} 0 & 0 \\ -1 & 0 \\ 1 & 0 \\ 0 & -1 \\ 0 & 1 \\ 0 & 0 \end{bmatrix}$$

$$a_{11} = \left[\begin{bmatrix} 0.5 & -1 & 0.67 & -0.33 & 0 & 0 \\ 0 & 0 & 0.33 & -0.67 & 1 & -0.5 \end{bmatrix} \begin{bmatrix} 0 & 0 \\ -1 & 0 \\ 1 & 0 \\ 0 & -1 \\ 0 & 1 \\ 0 & 0 \end{bmatrix} \frac{1}{EI} \right]$$

$$a_{11} = \frac{1}{EI} \begin{bmatrix} 1.67 & -0.33 \\ 0.33 & 1.67 \end{bmatrix}$$

$$(a_{11})^{-1} = EI \begin{bmatrix} 0.58 & 0.11 \\ -0.11 & 0.58 \end{bmatrix}$$

$$a_{10} = (b')^{T} \alpha (b^{o})$$

$$= \begin{bmatrix} 0.5 & -1 & 0.67 & -0.33 & 0 & 0 \\ 0 & 0 & 0.33 & -0.67 & 1 & -0.5 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \times \frac{1}{EI}$$

$$= \frac{1}{EI} \begin{bmatrix} 0.5 & -1 & 0.33 & 0 \\ 0 & 0 & -0.67 & -0.5 \end{bmatrix}$$

$$F' = -(a_{11})^{-1}(a_{10})(F^{o})$$

$$= -EI \begin{bmatrix} 0.58 & 0.11 \\ -0.11 & 0.58 \end{bmatrix} \frac{1}{EI} \begin{bmatrix} 0.5 & -1 & 0.33 & 0 \\ 0 & 0 & -0.67 & -0.5 \end{bmatrix} \times \begin{bmatrix} 9 \\ 7 \\ -10 \\ -6 \end{bmatrix}$$

$$F' = -\begin{bmatrix} (0.58 \times 0.05) & -0.58 & [(0.58 \times 0.33) + (0.11 \times -0.67)] & (0.11 \times 0.5) \\ (-0.11 \times 0.5) & 0.11 & [(-0.11 \times 0.33) + 0.58 \times -0.67] & (0.58 \times -0.5) \end{bmatrix} \begin{bmatrix} 9 \\ 7 \\ -10 \\ -6 \end{bmatrix}$$

$$F' = \begin{bmatrix} 0.29 & -0.58 & 0.12 & -0.055 \\ -0.055 & 0.11 & -0.42 & -0.29 \end{bmatrix} \begin{bmatrix} 9 \\ 7 \\ -10 \\ -6 \end{bmatrix}$$

$$F' = \begin{bmatrix} (0.29 \times 9) & (-0.58 \times 7) & (0.12 \times -10) & (-0.055 \times -6) \\ (-0.055 \times 9) & (0.11 \times 7) & (-0.42 \times -10) & (-0.29 \times -6) \end{bmatrix}$$

$$F' = \begin{bmatrix} 2.32 \\ -6.21 \end{bmatrix}$$

step 7: To find End Moments:

$$P = (b^{\circ})(F^{\circ}) + (b')(F') + (P^{\circ})$$

$$= \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 9 \\ 7 \\ -10 \\ -6 \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ -1 & 0 \\ 1 & 0 \\ 0 & -1 \\ 0 & 1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} 2.32 \\ -6.21 \end{bmatrix}$$

$$+ \begin{bmatrix} -9 \\ 9 \\ -16 \\ 16 \\ -6 \\ 6 \end{bmatrix}$$

$$= \begin{bmatrix} 9 \\ 7 \\ 0 \\ -10 \\ 0 \\ -6 \end{bmatrix} + \begin{bmatrix} 0 \\ -2.32 \\ 2.32 \\ 6.21 \\ -6.21 \\ 0 \end{bmatrix} + \begin{bmatrix} -9 \\ 9 \\ -16 \\ 16 \\ -6 \\ 6 \end{bmatrix}$$

$$P = \begin{bmatrix} 0 \\ +13.68 \\ -13.68 \\ 12.21 \\ -12.21 \\ 0 \end{bmatrix} \text{ kNm}$$

step 8: To find Free BMD

span,

$$AB = \frac{wl^2}{8}$$

$$= \frac{14 \times 3^2}{8} = 13.5 \text{ kNm}$$

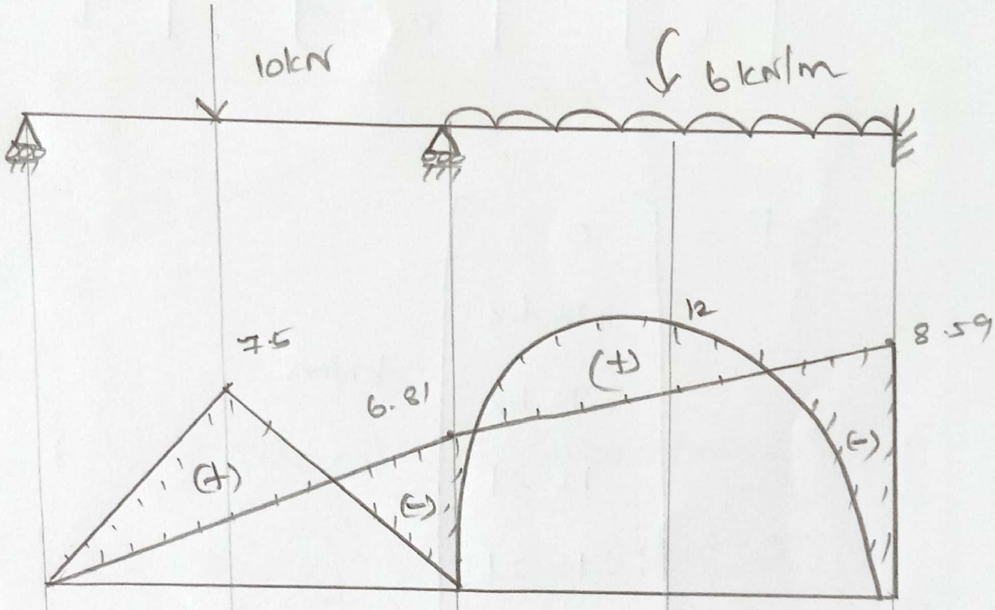
$$BC = \frac{wl^2}{8}$$

$$= \frac{12 \times 4^2}{8} = 24 \text{ kNm}$$

$$CD = \frac{wl}{4}$$

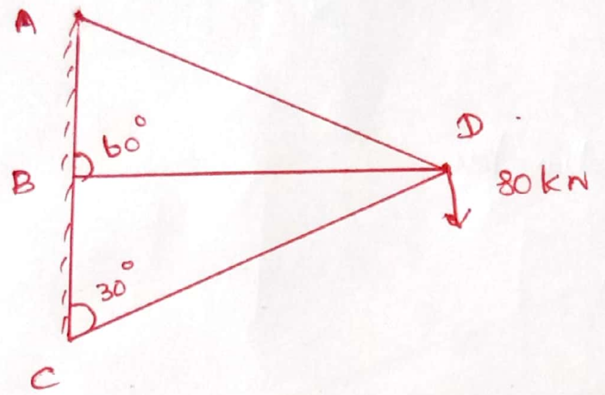
$$= \frac{16 \times 3}{4}$$

$$= 12\ kNm$$



4) Analysis of pin jointed Truss or Frame by Flexibility Method.

Take $\frac{AE}{l} = 1$ for all members.



step 1 :

Degree of Internal Indeterminancy

$I = m - 2j$ (for fixed truss)

$I = 3 - 2(1) = 1$

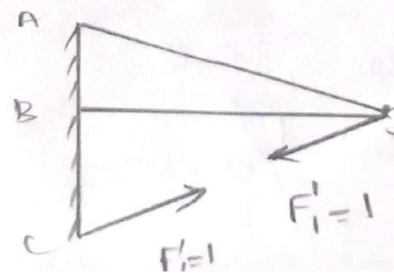
step 2 : To find F^0 :

$F^0 = 80$

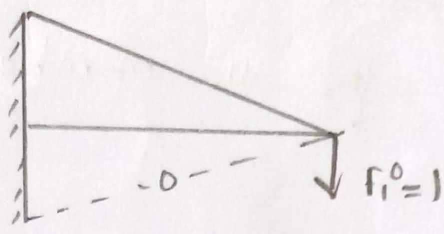
$I = 1$. Hence the given truss is internally indeterminate.

step 3 : Redundant system :

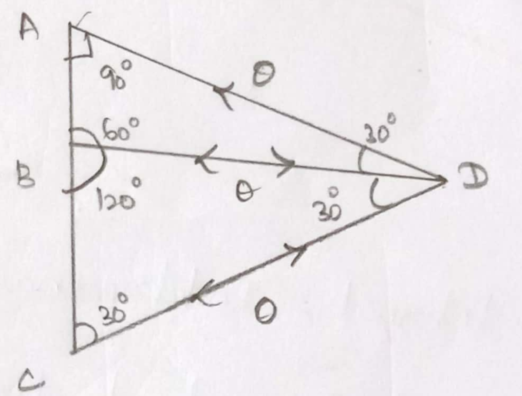
Let us treat the member CD as redundant.



system co-ordinate



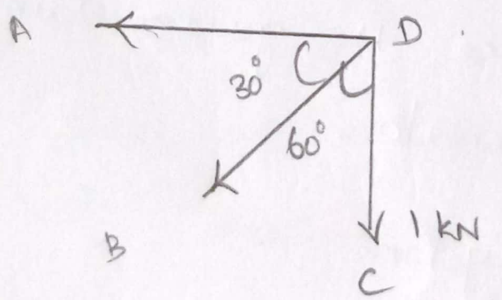
Element co-ordinates ;



step 4:

To find b^0 :-

Apply $f_1^0 = 1$



solving using method of Joints.

Joint D,

$$-1 - F_{DB} \cos 60^\circ = 0$$

$$-F_{DB} = 1/\cos 60^\circ = -2 \text{ kN}$$

$\Sigma H = 0$;

$$-FDA - FBD \cos 30^\circ = 0$$

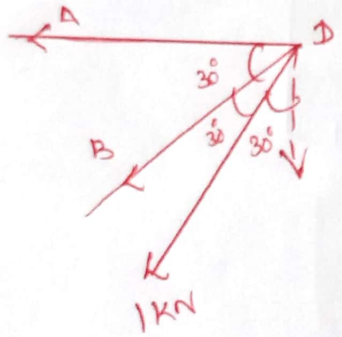
$$-FDA + 2 \cos 30^\circ = 0$$

$$FDA = 1.73 \text{ kN}$$

$$b^0 = \begin{bmatrix} 1.73 \\ -2 \\ 0 \end{bmatrix}$$

step 5 : To find b' :

Applying $F_1' = 1$



At joint D :

$$\Sigma V = 0$$

$$-F_{DB} \cos 60^\circ = 1 \cos 30^\circ$$

$$F_{DB} = 1.732 \text{ kN}$$

$\Sigma H = 0$;

$$-FDA - FDB \cos 30^\circ - \cos 60^\circ = 0$$

$$-FDA + 1.732 \cos 30^\circ - \cos 60^\circ = 0$$

$$F_{DA} = 1 \text{ kN}$$

$$b' = \begin{bmatrix} 1 \\ 1.732 \\ 1 \end{bmatrix}$$

Step 6:

To find α:

$$\alpha = {}^{l}/_{AE} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$ for 3 elements

Given $AE/_{l} = 1$

$$\alpha = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Step 7: To find F':

$$F' = (-a_{11})^{-1} (a_{10})(F^{\circ})$$

$$a_{11} = (b')^{T} (\alpha) (b')$$

$$a_{10} = (b')^{T} (\alpha) (b^{\circ})$$

$$a_{11} = \begin{bmatrix} 1.73 & -2 & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1.73 \\ -2 \\ 0 \end{bmatrix}$$

$$a_{11} = [4.99]$$

$$-(a_{11})^{-1} = -[0.2]$$

$$a_{10} = \begin{bmatrix} 1 & -1.732 & 1 \end{bmatrix} \begin{bmatrix} 1.73 \\ -2 \\ 0 \end{bmatrix}$$

$$a_{10} = \begin{bmatrix} 5.194 \end{bmatrix}$$

$$F_1' = (-a_{11})^{-1} (a_{10})(F^0)$$

$$F' = -0.2 \times 5.2 \times 80 = -83.2 \text{ kN}$$

Step 8: To find "P".

$$P = (b^0)(F^0) + (b')(F') + p^0$$

$$= \begin{bmatrix} 1.732 \\ -2 \\ 0 \end{bmatrix} \begin{bmatrix} 80 \end{bmatrix} + \begin{bmatrix} 1 \\ -1.732 \\ 1 \end{bmatrix} \begin{bmatrix} -83.2 \end{bmatrix} + 0$$

$$P = \begin{bmatrix} 55.4 \\ -16 \\ -83.2 \end{bmatrix} \text{ kN}$$

F_{DA} = 55.4 kN (Tension)

F_{DB} = −16 kN (compression)

F_{DC} = −83.2 kN (compression)

5. Stiffness Matrix Method

In this stiffness matrix method, nodal displacement are treated as basic unknowns for the solution of indeterminate structures.

Nodal displacements → basic unknowns

Element and Global stiffness matrix:

i) Element stiffness matrix

It is denoted by (K) for beam or frame

$$K = \frac{2EI}{l}\begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}$$ for a element

For truss.

$$K = \frac{AE}{l}\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$ for three elements.

After the continuous is discretized with the desired element shapes. The element stiffness matrix are formulated. These can be done by using either equlibrium conditions (or) a suitable variational principle such as minimum potential energy principle.

Global stiffness matrix:

After the element stiffness matrix in global coordinates are formed they are assembled to form the overall(Global) stiffness matrix.

The final global finite equation for the complete structure can be written as,

$$(K) = (\beta)^T (k) (\beta)$$

where,

$[\beta] \to$ Transformation matrix

$[k] \to$ Element stiffness matrix

co-ordinate Transformation:

For specifying a configuring of a system a certain number of independent are necessary.

The least number of independent co-ordinates that are needed to specify the configuration is known as Generalized co-ordinates.

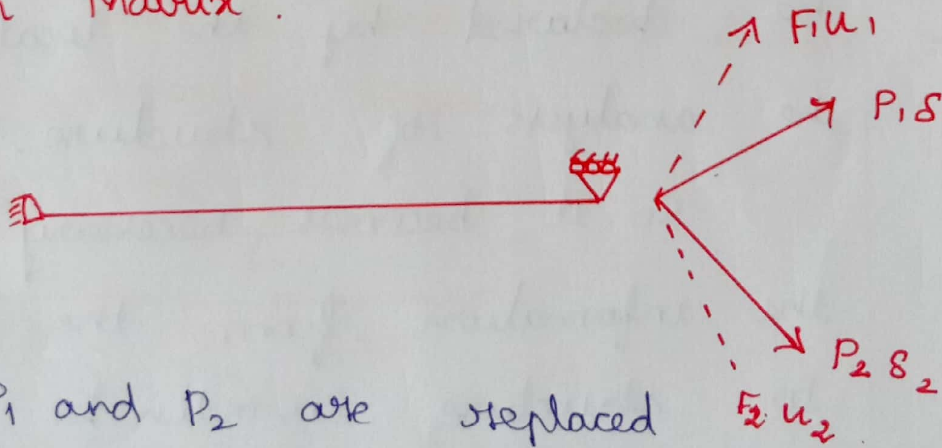
System or Global coordinates:

In problem solving is used to define a co-ordinate system dealing with the entire structure.

Coordinates are assigned to locations especially nodes where loads are likely to act.

This kind of co-ordinates are called System coordinates or Global coordinates.

Rotation Matrix:



In P_1 and P_2 are replaced by a set of statically equivalent forces, F_1 and F_2.

$$F_1 = P_1 \cos\alpha - P_2 \sin\alpha$$

$$F_2 = P_1 \sin\alpha + P_2 \cos\alpha$$

$$[F] = [T][P]$$

where,

$[T] \rightarrow$ Transformation matrix

$$[T] = \begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix}$$

$$[P] = \begin{bmatrix} P_1 \\ P_2 \end{bmatrix}$$

Transformation of stiffness matrix:

The coordinate system dealing with entire system is called Global coordinate system or system co-ordinates has to be declared by the local coordinates in the analysis of structure.

So it becomes necessary to transform the information from the member coordinate to structure co-ordinates and wise versa

such a matrix used for transformation of information is transformation matrix.

Transformation of load vector and displacement vector:

Displacement vector

It is denoted by $\Delta \Rightarrow \Delta = [K^{-1}][P]$

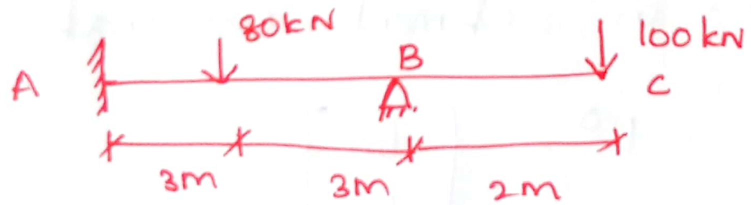
$[K^{-1}]$ = Inverse matrix for global stiffness

$[P]$ = load vector

Load Vector:

It is denoted by "P" $\Rightarrow [P] = [\Delta][K]$

Analyse the beam shown in the figure by stiffness matrix method and draw the bending moment diagram.



step 1: To find kinematic Indeterminancy

since the only portable independent joint displacement happens to be θ_B.

$K \cdot I = 1$

step 2 :

system coordinate 

Element coordinate 

step 3 : To find FEM(P^0)

$$M_{FAB} = -WL/8$$

$$= -\frac{80 \times 6}{8}$$

$$= -60 \text{ kNm}$$

$$M_{FAB} = WL/8$$

$$= 60 \text{ kN m}$$

$$M_{BC} = 100 \times 2 = 200 \text{ kNm}$$

forces in element coordinates

$$P^0 = \begin{bmatrix} -60 \\ 60 \end{bmatrix}$$

step 4 : Transformation matrix

$$[B] = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

step 5 To find F :

$$[F] = [F^c] - [F^F]$$

$$[F] = 140$$

step 6 : To find element stiffness matrix

$$K = \frac{2EI}{l}\begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}$$

$$= \frac{2EI}{6}\begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}$$

$$K = \frac{EI}{3}\begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}$$

step 7 : To find Global system matrix

$$[K] = [B^T][k][B]$$

$$= \begin{bmatrix} 0 & 1 \end{bmatrix} \frac{EI}{3}\begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}\begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$$= \frac{EI}{3}\begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix}\begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$$= EI/3 \begin{bmatrix} 2 \end{bmatrix} = \frac{2EI}{3}$$

steps 8 : To find system displacement (U)

$$U = (k^{-1})(F)$$

$$U = 3/_{2EI} \times 140 = \frac{210}{EI}$$

step 9 : To find element displacement (δ)

$$\delta = (\beta)(U)$$

$$= \begin{bmatrix} 0 \\ 1 \end{bmatrix} \begin{bmatrix} 210/EI \end{bmatrix}$$

step 10 : To find (P') forces :

$$P' = K\delta$$

$$= \frac{EI}{3} \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix} \frac{1}{EI} \begin{bmatrix} 0 \\ 210 \end{bmatrix}$$

$$= \frac{1}{3} \begin{bmatrix} 210 \\ 420 \end{bmatrix}$$

$$P' = \begin{bmatrix} 70 \\ 140 \end{bmatrix}$$

step 11 : To find P

$$P = P' + P^{o}$$

$$= \begin{bmatrix} 70 \\ 140 \end{bmatrix} + \begin{bmatrix} -60 \\ 60 \end{bmatrix}$$

$$P = \begin{bmatrix} 10 \\ 200 \end{bmatrix} \text{ kNm}$$

$M_{AB} = 10$ kNm

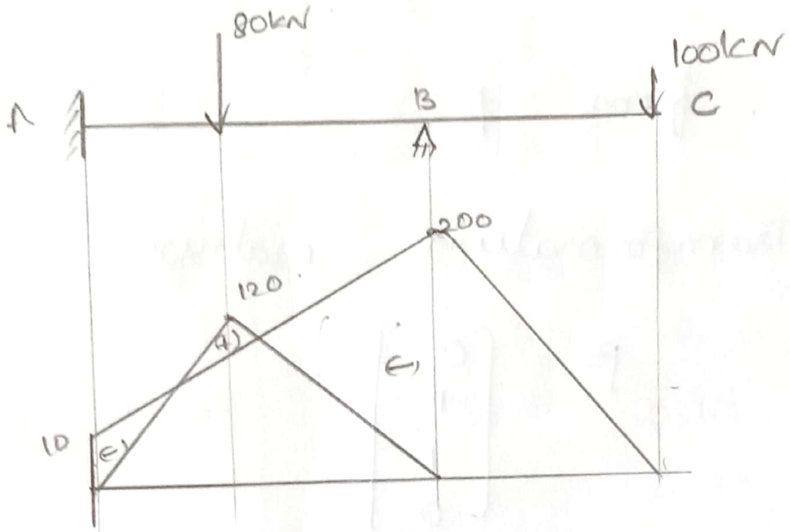
$M_{BA} = 200$ kNm

$M_{BC} = -200$ kNm

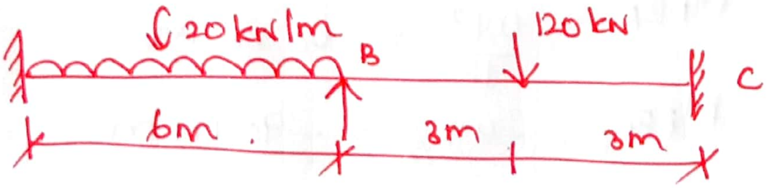
Step 12 : To find free BM

$AB = WL/8$

$= 80 \times 6/8 = 120$ kN m



Analyse the continuous beam loaded as shown in figure by stiffness matrix method



Step 1 :

To find kinematic Indeterminancy

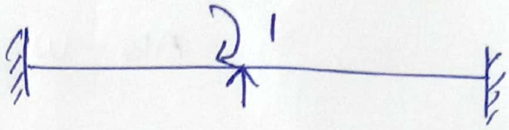
The structure is kinematically indeterminate to first degree ; ie) θ_B is unknown

Joint independent displacements.

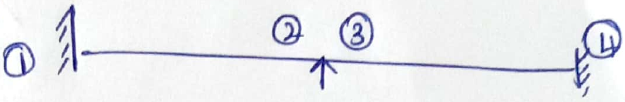
$K.I = 1$

Step 2 :

i) System coordinate :



ii) Element coordinate



To form β :

Transformation matrix :

$$\beta = \begin{bmatrix} 0 \\ 1 \\ 1 \\ 0 \end{bmatrix}$$

Step 3 : To find FEM (P°)

$$M_{FAB} = \frac{-WL^2}{12} = -60 \text{ kN m}$$

$$M_{FBA} = \frac{WL^2}{12} = 60 \text{ kN m}$$

$$M_{FBC} = -WL/8 = -90 \text{ kN m}$$

$$M_{FCB} = WL/8 = 90 \text{ kNm}.$$

$$P^{o} = \begin{bmatrix} -60 \\ 60 \\ -90 \\ 90 \end{bmatrix} \text{ kNm}.$$

Step 4 : To find (F) :

$$(F) = [F^{C}] - [F^{F}]$$

$$(F) = 0 - (60 - 90) = 30$$

step 5: To find element stiffness matrix (k)

$$k = \frac{2EI}{l}\begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix} \text{ for one element}$$

$$= \frac{2EI}{l}\begin{bmatrix} 2 & 1 & 0 & 0 \\ 1 & 2 & 0 & 0 \\ 0 & 0 & 2 & 1 \\ 0 & 0 & 1 & 2 \end{bmatrix}$$

$$k = EI\begin{bmatrix} 0.67 & 0.33 & 0 & 0 \\ 0.33 & 0.67 & 0 & 0 \\ 0 & 0 & 0.67 & 0.33 \\ 0 & 0 & 0.33 & 0.67 \end{bmatrix}$$

step 6: To find K:

$$[K] = [B^T][k][B]$$

$$= \begin{bmatrix} 0 & 1 & 1 & 0 \end{bmatrix} EI \begin{bmatrix} 0.67 & 0.33 & 0 & 0 \\ 0.33 & 0.67 & 0 & 0 \\ 0 & 0 & 0.67 & 0.33 \\ 0 & 0 & 0.33 & 0.67 \end{bmatrix}\begin{bmatrix} 0 \\ 1 \\ 1 \\ 0 \end{bmatrix}$$

$$= \begin{bmatrix} 0.33 & 0.67 & 0.67 & 0.33 \end{bmatrix}\begin{bmatrix} 0 \\ 1 \\ 1 \\ 0 \end{bmatrix} EI$$

$$K = EI\,(1.34)$$

step 7 : To find U :

$$U = [K^{-1}][F]$$

$$= \frac{1}{E} \times \frac{1}{1.34} \times 30$$

$$U = 22.39 / EI$$

step 8 : To find δ :

$$\delta = (\beta)(U)$$

$$= \begin{bmatrix} 0 \\ 1 \\ 1 \\ 0 \end{bmatrix} \left[\frac{22.39}{EI} \right]$$

$$\delta = \frac{1}{EI} \begin{bmatrix} 0 \\ 22.39 \\ 22.39 \\ 0 \end{bmatrix}$$

step 9 : To find (P') :

$$(P') = (K)(\delta)$$

$$= EI \begin{bmatrix} 0.67 & 0.33 & 0 & 0 \\ 0.33 & 0.67 & 0 & 0 \\ 0 & 0 & 0.67 & 0.33 \\ 0 & 0 & 0.33 & 0.67 \end{bmatrix} \times \frac{1}{EI} \begin{bmatrix} 0 \\ 22.39 \\ 22.39 \\ 0 \end{bmatrix}$$

step

$$(P) = \begin{bmatrix} 0.33 \times 22.39 \\ 0.67 \times 22.39 \\ 0.67 \times 22.39 \\ 0.33 \times 22.39 \end{bmatrix}$$

$$= \begin{bmatrix} 7.39 \\ 15 \\ 15 \\ 7.39 \end{bmatrix}$$

step 10: To find forces (P):

$$[P] = [P'] + [P^{o}]$$

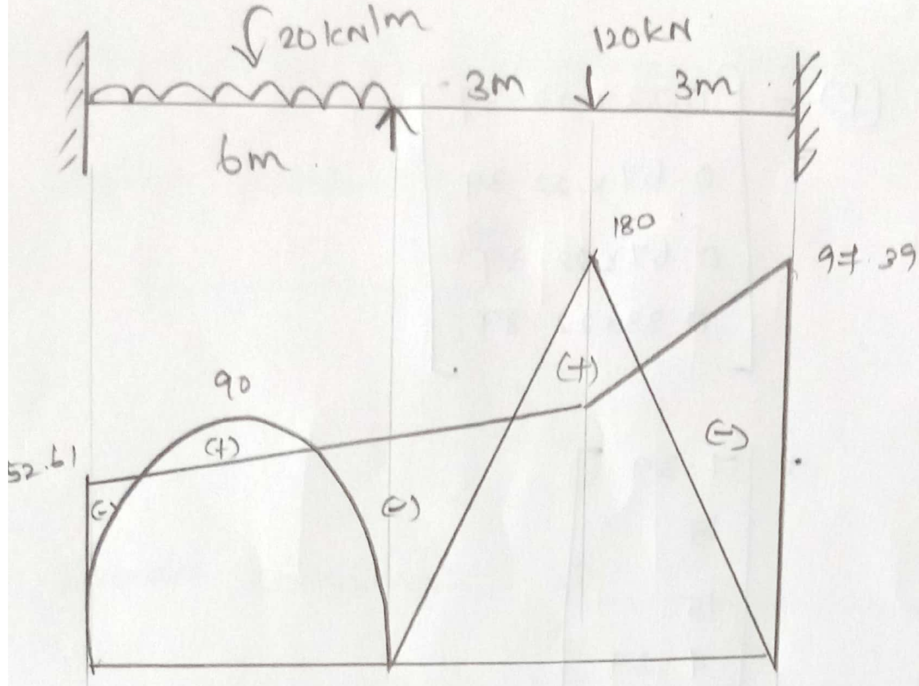
$$= \begin{bmatrix} 7.39 \\ 15 \\ 15 \\ 7.39 \end{bmatrix} + \begin{bmatrix} -60 \\ 60 \\ -90 \\ 90 \end{bmatrix}$$

$$[P] = \begin{bmatrix} -52.61 \\ 75 \\ -75 \\ 97.39 \end{bmatrix}$$

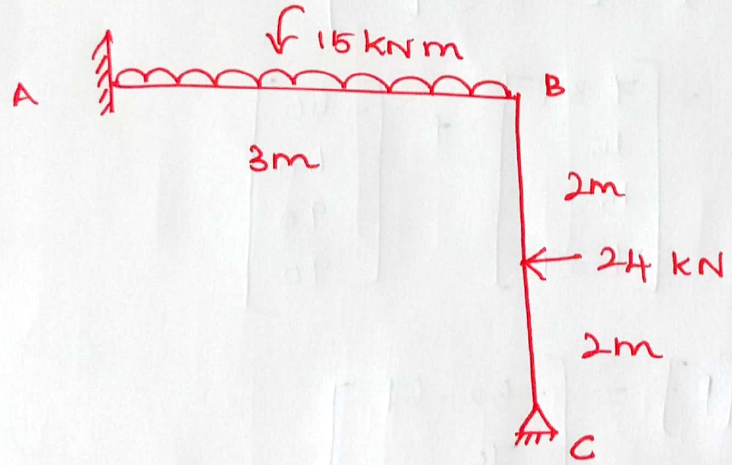
step 11: To find Free BMD

for span $AB = wl^2/8 = 90$ kN m

for span $BC = WL/4 = 180$ kN m



Analyse the frame loaded as shown in figure by stiffness matrix methods.



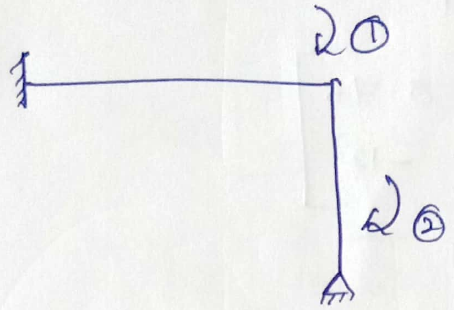
step 1 : To find kinematic indeterminancy.

Kinematic Indeterminancy = 2

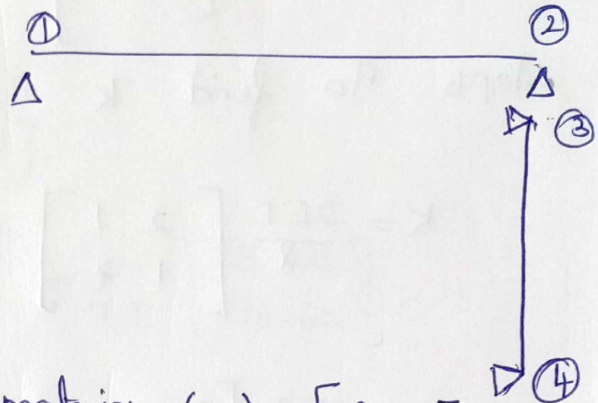
∴ Since it has kinematically indeterminate in degree and rotations at B and C are kinematic Indeterminacies.

Step 2: To find system coordinate

System coordinate:



Element coordinate



Step 3:

To find β:

Transformation matrix $(\beta) = \begin{bmatrix} 0 & 0 \\ 1 & 0 \\ 1 & 0 \\ 0 & 1 \end{bmatrix}$

Step 4: To find FEM (P^0)

$M_{FAB} = -WL^2/12 = -11.25$ kNm

$M_{FBA} = WL^2/12 = 11.25$ kNm

$M_{FBC} = -WL/8 = -12$ kNm

$M_{FCB} = WL/8 = 12$ kNm.

$$P^0 = \begin{bmatrix} -11.25 \\ 11.25 \\ -12 \\ 12 \end{bmatrix}$$

step4: To find F

$$[F] = F^c - F^F$$

$$= 0 - \begin{bmatrix} 0.75 \\ -12 \end{bmatrix}$$

$$[F] = \begin{bmatrix} 0.75 \\ -12 \end{bmatrix}$$

step5: To find k:

$$k = \frac{2EI}{l}\begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}$$

$$= EI\begin{bmatrix} 1.33 & 0.67 & 0 & 0 \\ 0.67 & 1.33 & 0 & 0 \\ 0 & 0 & 1.33 & 0.67 \\ 0 & 0 & 0.67 & 1.33 \end{bmatrix}$$

step6: To find Global stiffness matrix $[K^{-1}]$

$$[K] = [\beta^T][k][\beta]$$

$$= \begin{bmatrix} 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} EI \begin{bmatrix} 1.33 & 0.67 & 0 & 0 \\ 0.67 & 1.33 & 0 & 0 \\ 0 & 0 & 1.33 & 0.67 \\ 0 & 0 & 0.67 & 1.33 \end{bmatrix}\begin{bmatrix} 0 & 0 \\ 1 & 0 \\ 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$K = EI\begin{bmatrix} 2.33 & 0.5 \\ 0.5 & 1 \end{bmatrix} \quad K^{-1} = \frac{1}{EI}\begin{bmatrix} 0.48 & -0.24 \\ -0.24 & 1.12 \end{bmatrix}$$

step 7 To find U

$$U = [K^{-1}][F]$$

$$= \frac{1}{EI}\begin{bmatrix} 0.48 & -0.24 \\ -0.24 & 1.12 \end{bmatrix}\begin{bmatrix} 8.75 \\ -12 \end{bmatrix}$$

$$= \frac{1}{EI}\begin{bmatrix} 3.24 \\ 13.62 \end{bmatrix}$$

step 8 : To find S

$$S = [B][U]$$

$$= \begin{bmatrix} 0 & 0 \\ 1 & 0 \\ 1 & 0 \\ 0 & 1 \end{bmatrix} \times \frac{1}{EI}\begin{bmatrix} 3.24 \\ 13.62 \end{bmatrix}$$

step 9 : To find P

$$P = [k][S] + P^{\circ}$$

$$= EI\begin{bmatrix} 0.67 & 0.33 & 0 & 0 \\ 0.33 & 0.67 & 0 & 0 \\ 0 & 0 & 1.33 & 0.67 \\ 0 & 0 & 0.67 & 1.33 \end{bmatrix} \times \frac{1}{EI}\begin{bmatrix} 0 \\ -55.71 \\ -55.71 \\ 12.86 \end{bmatrix}$$

$$+ \begin{bmatrix} 11.25 \\ 11.25 \\ -12 \\ 12 \end{bmatrix}$$

$$= \begin{bmatrix} -78.57 \\ 22.86 \\ -22.86 \\ 0 \end{bmatrix}$$

Scanned with OKEN Scanner

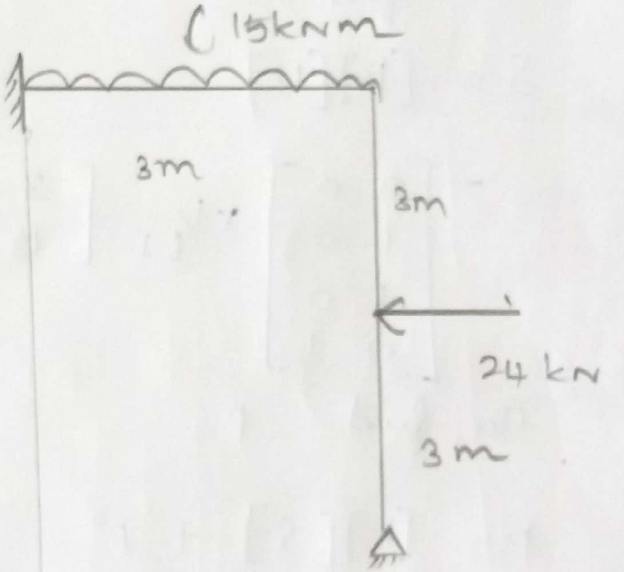
step 10 : To find free BMD

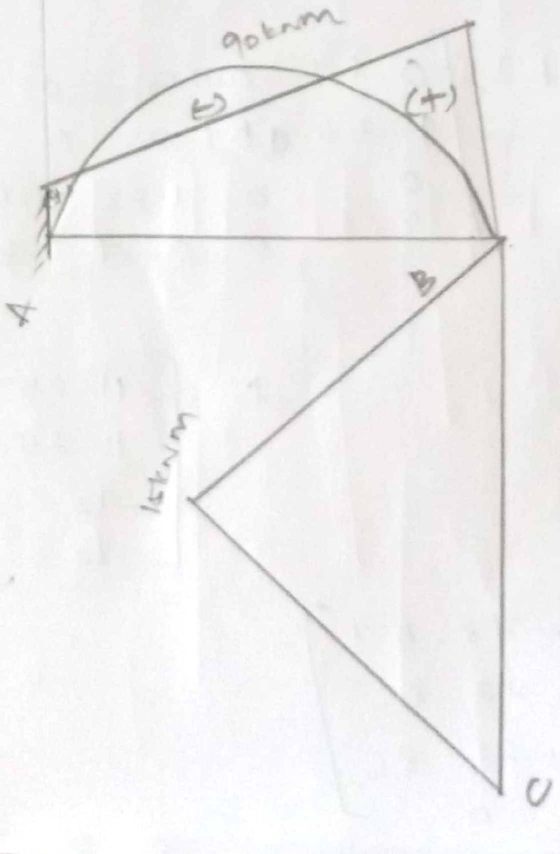
span AB $= \frac{wL^2}{8}$

$= 90$ kN m

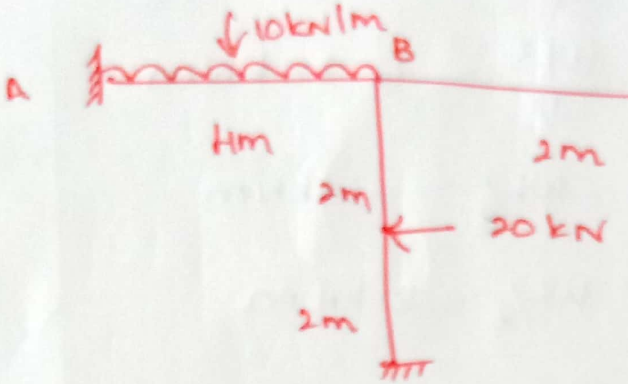
BC $= wL/_8$

$= 15$ kN m.





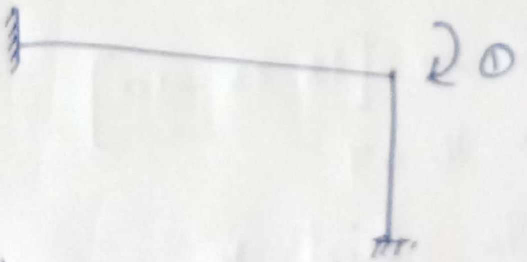
Analyse the frame loaded as shown in figure by stiffness matrix method.



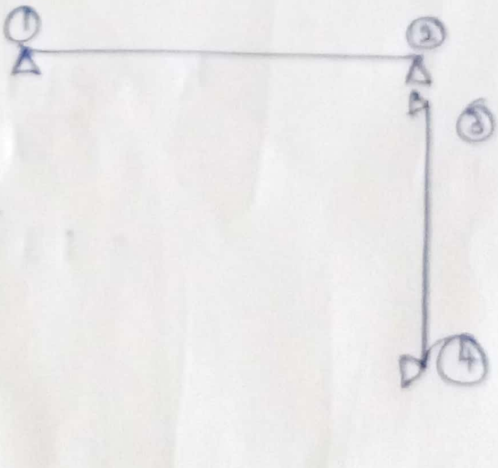
step 1: To find kinematic Indeterminancy

kinematic Indeterminancy = 1

step 2: To find i) system co-ordinates



ii) Element co-ordinate



step 3: To form β:

$$\beta = \begin{bmatrix} 0 \\ 1 \\ 1 \\ 0 \end{bmatrix}$$

step4 : To find FEM (P^o)

$$M_{FAB} = \frac{-wL^2}{12} = -13.33 \text{ kNm}$$

$$M_{FBA} = \frac{wL^2}{12} = 13.33 \text{ kNm}$$

$$M_{FBC} = -wL/_8 = -10 \text{ kNm}$$

$$M_{FCB} = wL/_8 = 10 \text{ kNm}$$

$$P^o = \begin{bmatrix} -13.33 \\ 13.33 \\ -10 \\ 10 \end{bmatrix}$$

step 5 : To find F

$$F = F^C - F^F$$

$$= 20 - [13.33 \quad -10]$$

$$F = 16.67$$

step 6 : To find k

$$k = \frac{2EI}{l} \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}$$

$$= EI \begin{bmatrix} 1 & 0.5 & 0 & 0 \\ 0.5 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0.5 \\ 0 & 0 & 0.5 & 1 \end{bmatrix}$$

Step 6: To find Global stiffness matrix k

$$k = (\beta)^T (k)(\beta)$$

$$= \begin{bmatrix} 0 & 1 & 1 & 0 \end{bmatrix} \times EI \begin{bmatrix} 1 & 0.5 & 0 & 0 \\ 0.5 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0.5 \\ 0 & 0 & 0.5 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \\ 1 \\ 0 \end{bmatrix}$$

$$k = EI \begin{bmatrix} 2.33 & 0.5 \\ 0.5 & 1 \end{bmatrix}$$

$$k^{-1} = \frac{1}{EI} \begin{bmatrix} 0.48 & -0.24 \\ -0.24 & 1.12 \end{bmatrix}$$

Step 7: To find U

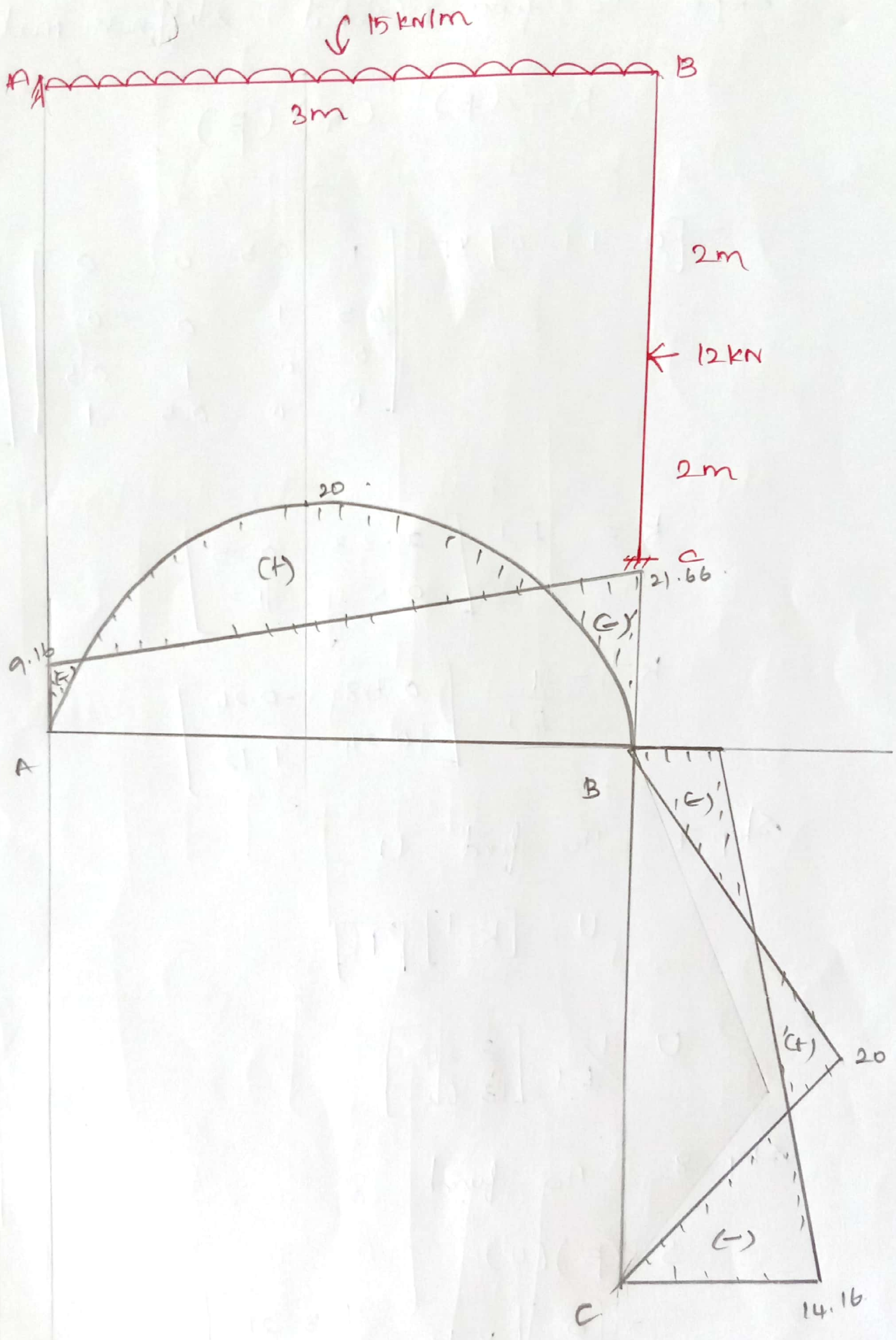
$$U = [k^{-1}][F]$$

$$U = \frac{1}{EI} \begin{bmatrix} 3.24 \\ 13.62 \end{bmatrix}$$

Step 8: To find δ

$$\delta = (\beta)(U)$$

$$= \begin{bmatrix} 0 \\ 1 \\ 1 \\ 0 \end{bmatrix} \times \frac{1}{EI} \begin{bmatrix} 3.24 \\ 13.62 \end{bmatrix}$$



15 kN/m
A
B
3m
2m
12 kN
2m
C
20
(+)
21.66
(-)
9.16
(-)
A
B
(-)
(+)
20
(-)
C
14.16